

# 堆和优先队列

胡船长

初航我带你，远航靠自己

# 本章题目

1-应试. Leetcode-703 : 数据流中的第 K 大元素

2-校招. Leetcode-295 : 数据流的中位数

3-校招. Leetcode-23 : 合并 K 个升序链表

4-校招. Leetcode-264 : 丑数 II

5-校招. HZOJ-284 : 超市卖货

6-竞赛. HZOJ-285 : 序列 M 小和

7-竞赛. HZOJ-289 : 生日礼物

# 本期内容

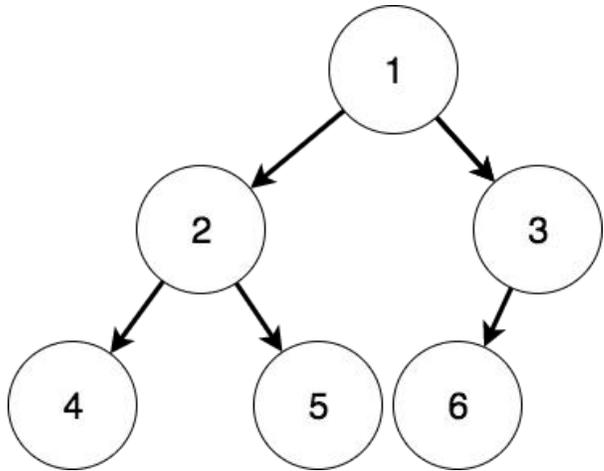
- 一. 堆与优先队列
- 二. 堆排序与线性建堆法
- 三. 优化：哈夫曼编码

# 一. 堆与优先队列

# 二叉树：完全二叉树

## 完全二叉树

(complete binary tree)



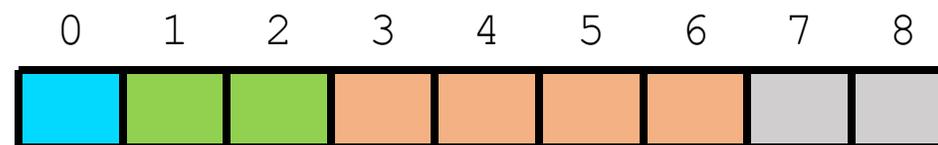
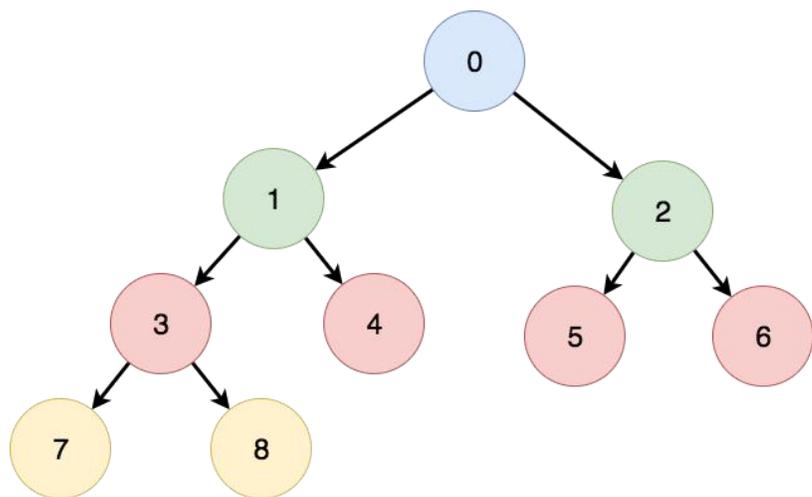
### 1、编号为 $i$ 的子节点：

左孩子编号： $2 * i$

右孩子编号： $2 * i + 1$

### 2、可以用连续空间存储（数组）

# 二叉树：完全二叉树

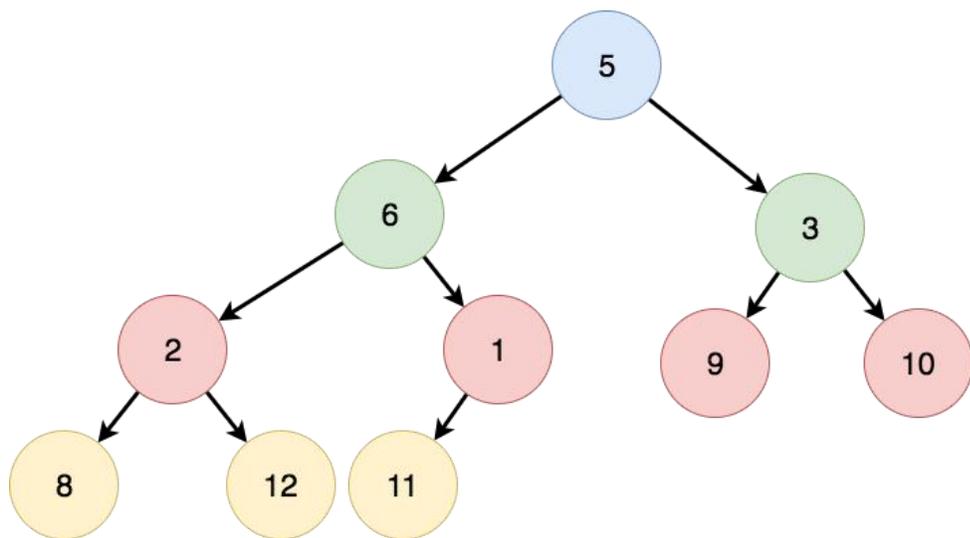


# 二叉树：完全二叉树

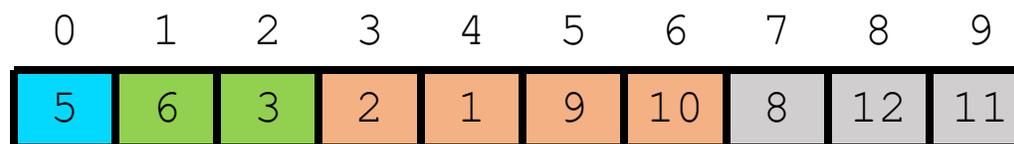
画出以下数组代表的完全二叉树

0	1	2	3	4	5	6	7	8	9
5	6	3	2	1	9	10	8	12	11

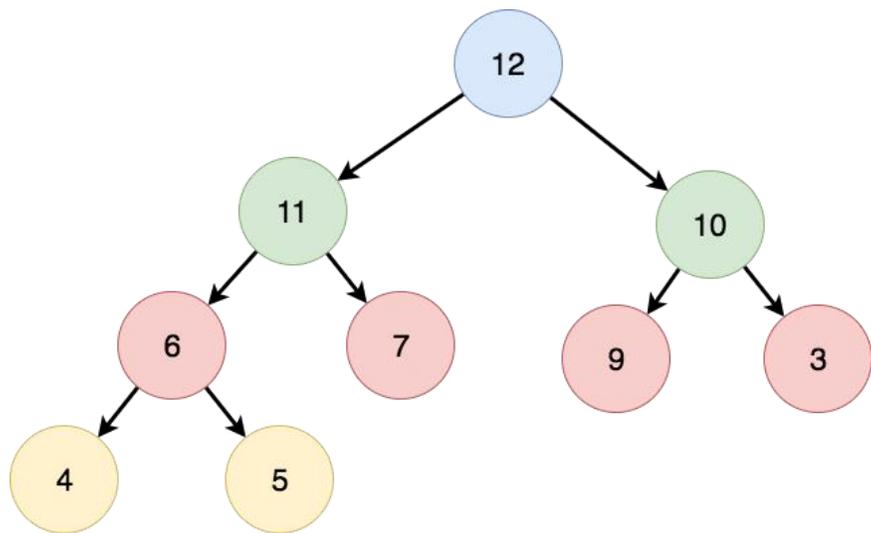
# 二叉树：完全二叉树



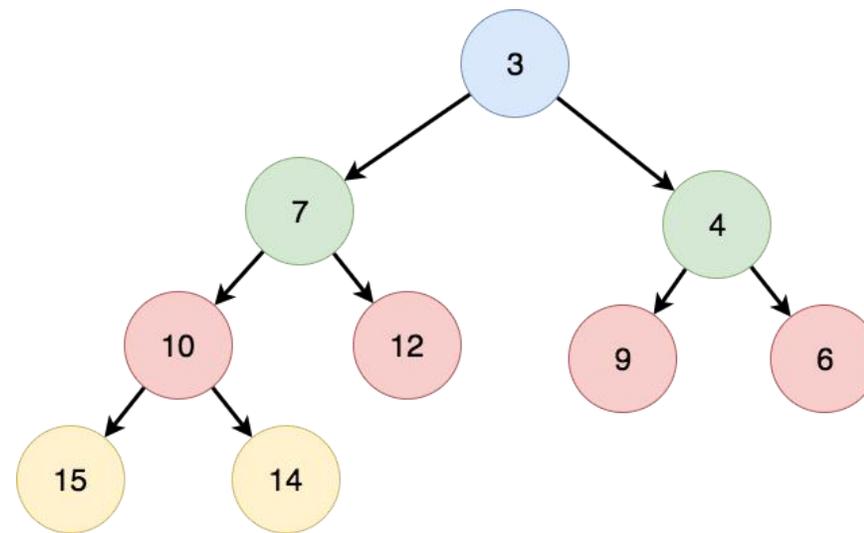
画出以下数组代表的完全二叉树



# 堆：结构讲解

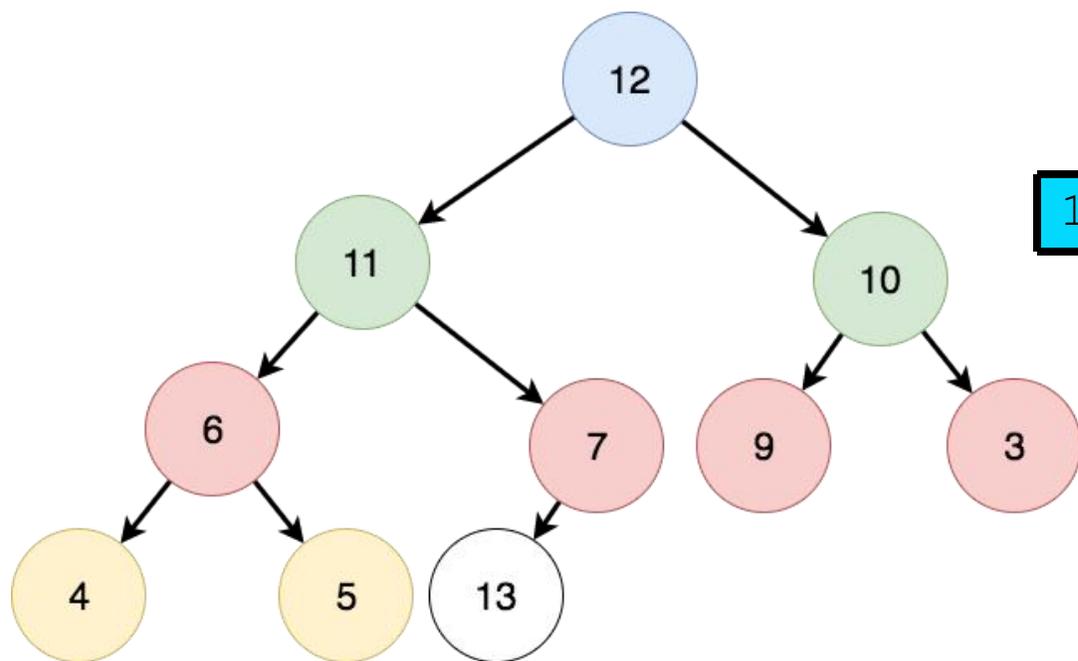


大顶堆

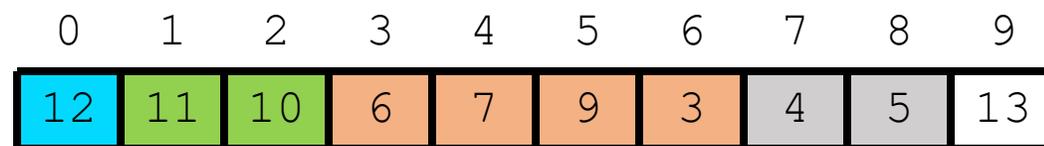


小顶堆

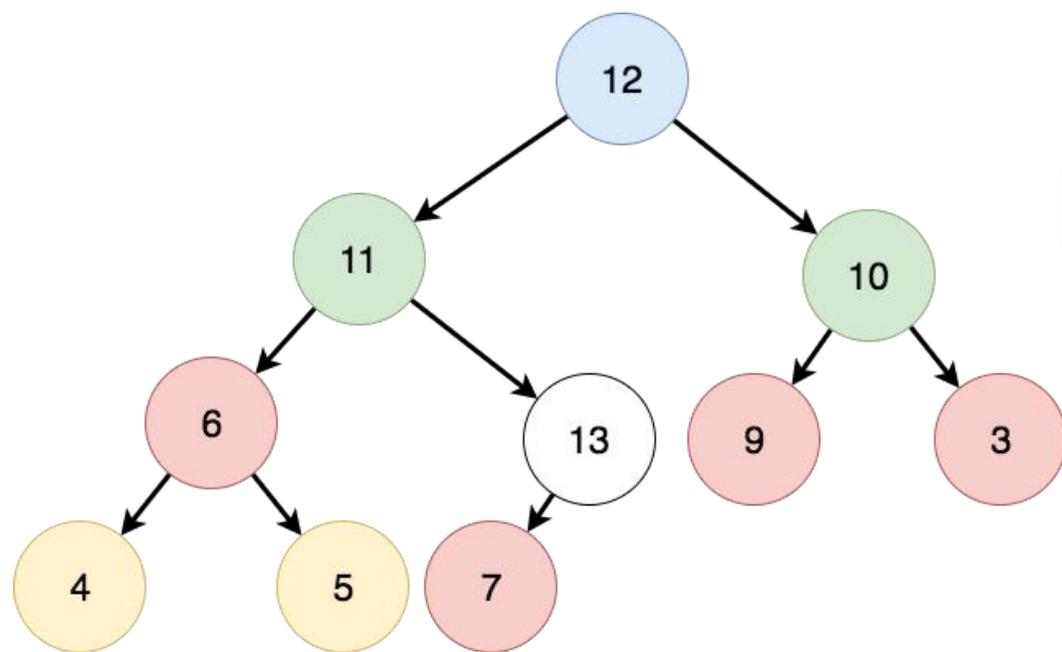
# 堆：尾部插入调整



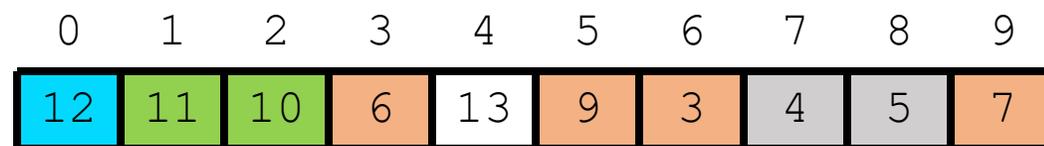
大顶堆



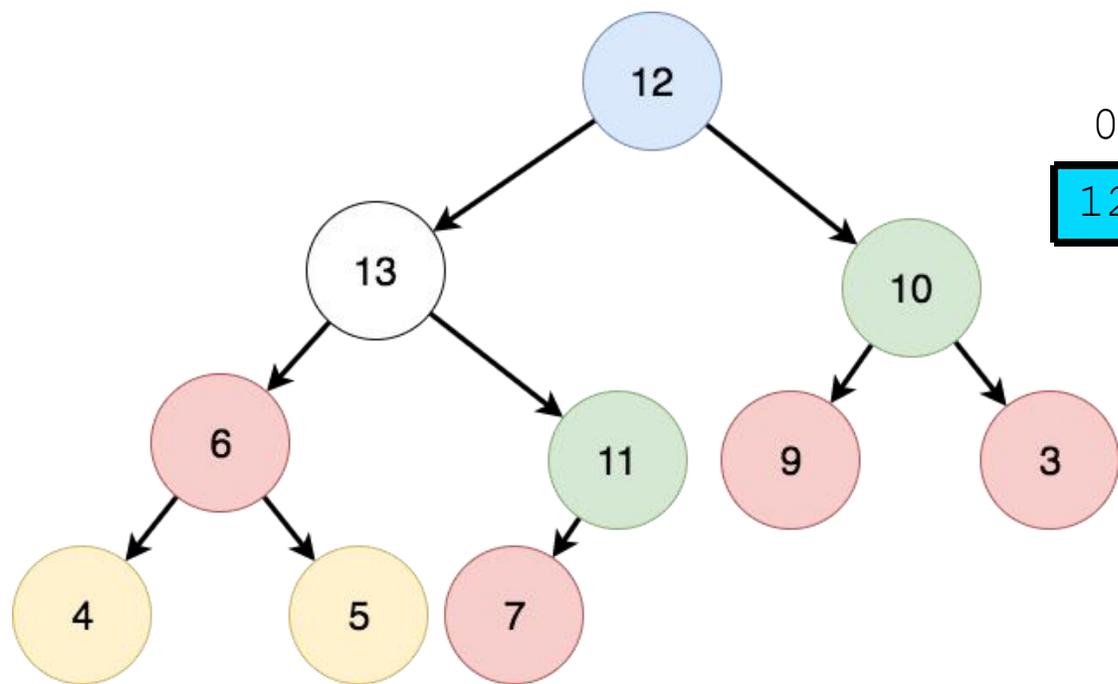
# 堆：尾部插入调整



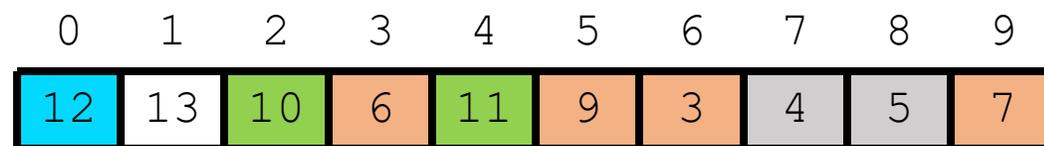
大顶堆



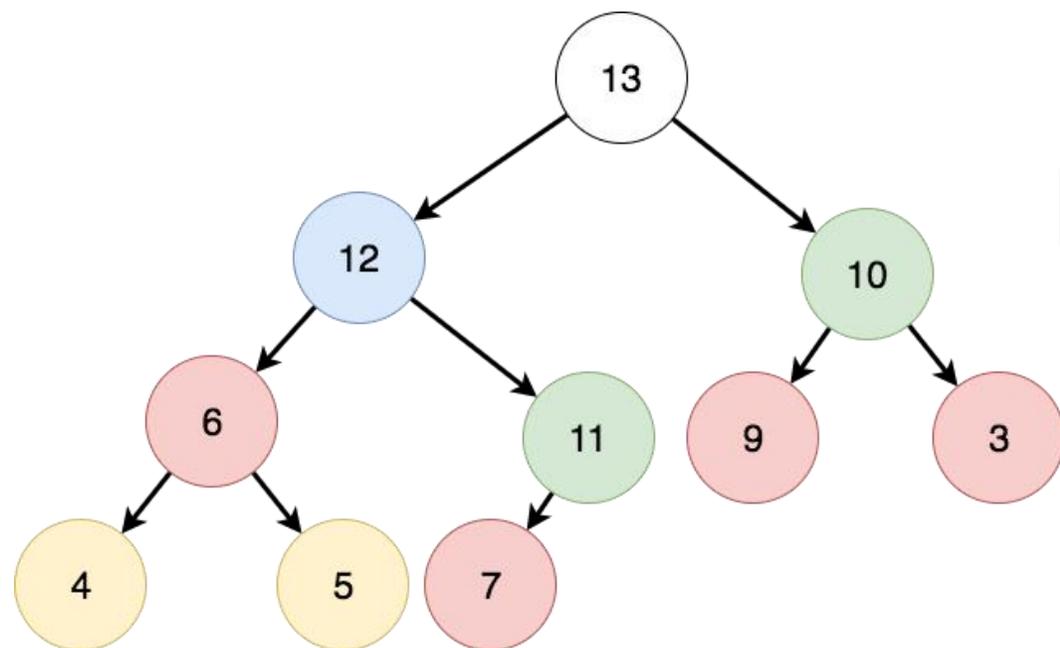
# 堆：尾部插入调整



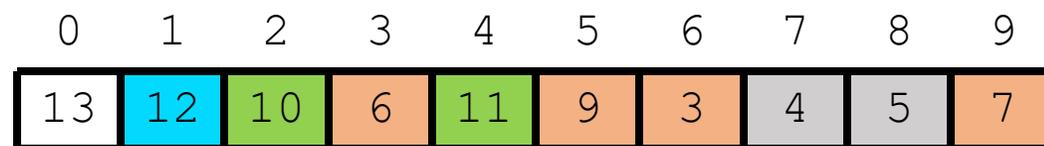
大顶堆



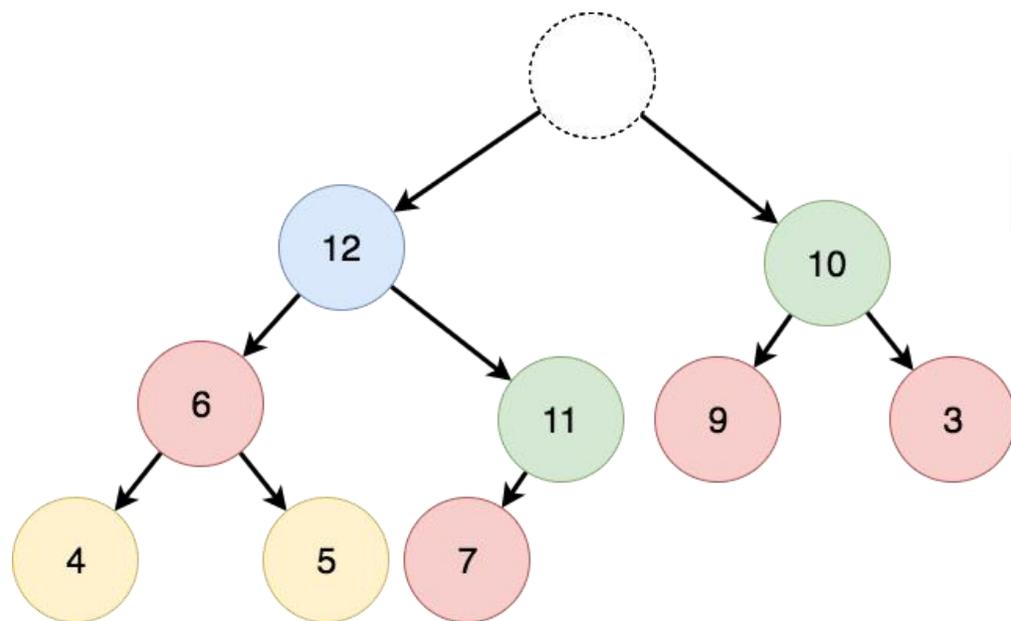
# 堆：尾部插入调整



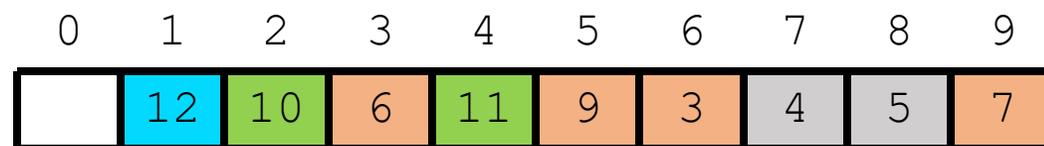
大顶堆



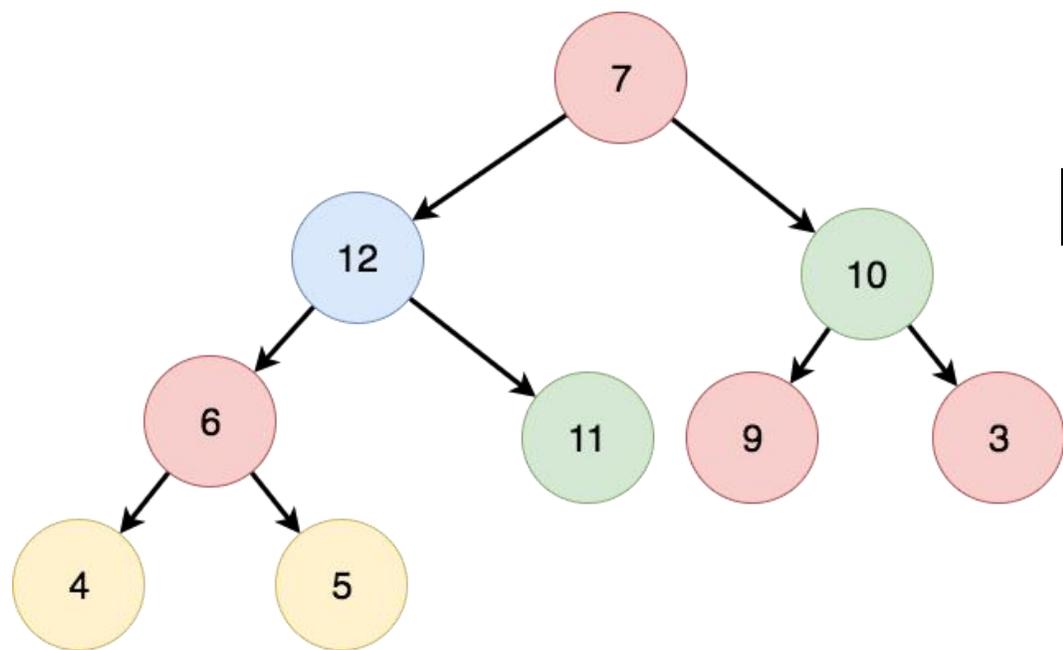
# 堆：头部弹出调整



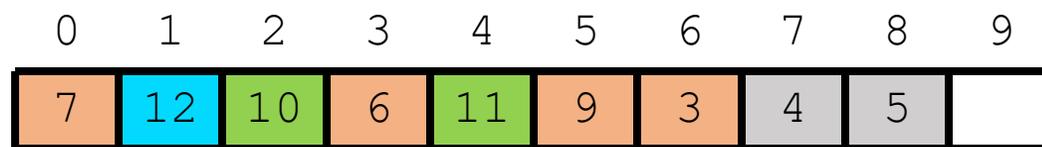
大顶堆



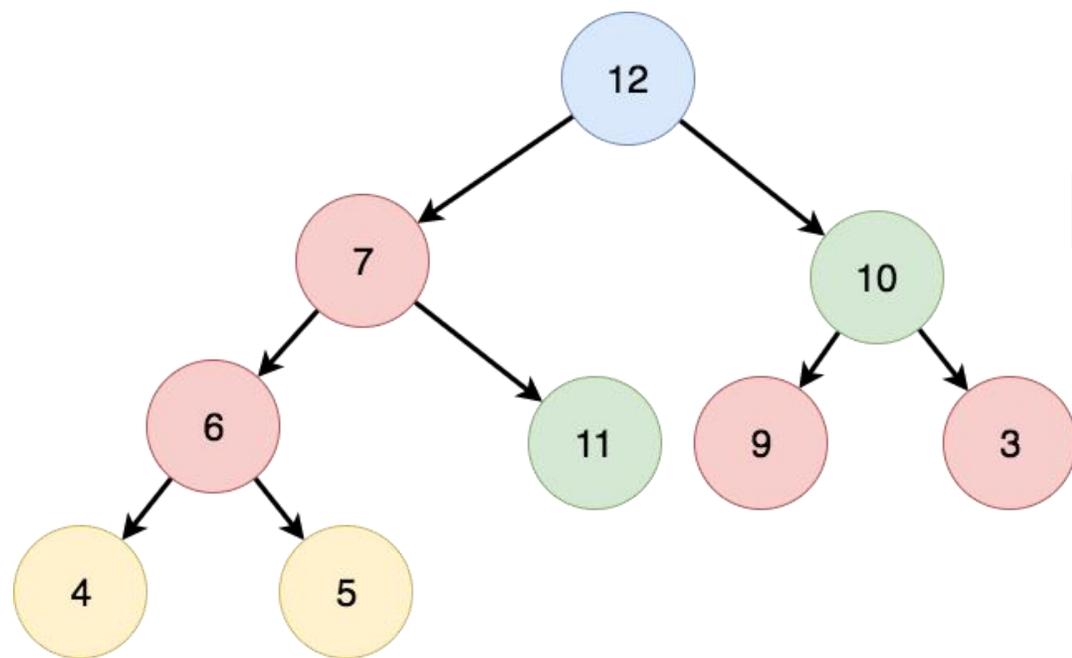
# 堆：头部弹出调整



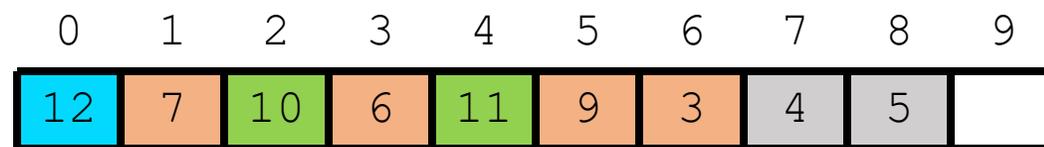
大顶堆



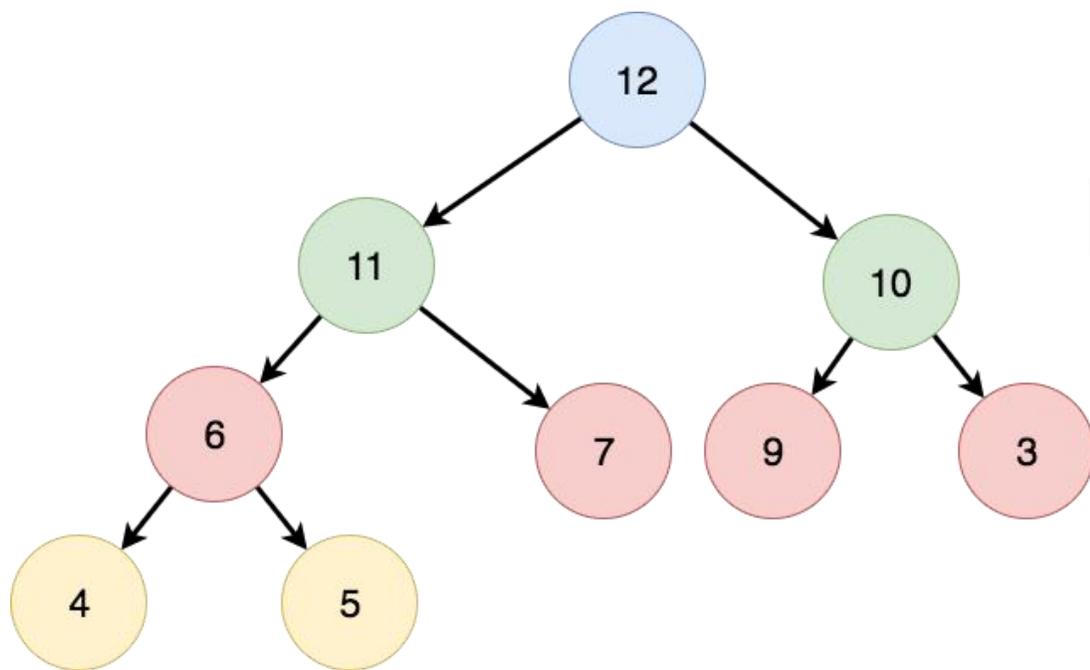
# 堆：头部弹出调整



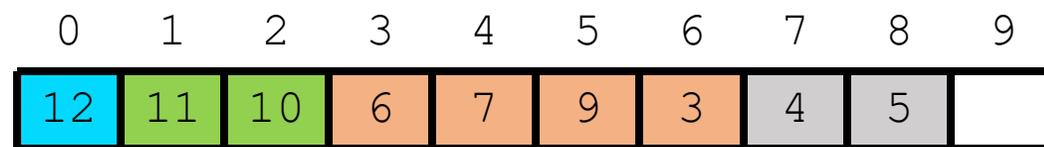
大顶堆



# 堆：头部弹出调整



大顶堆



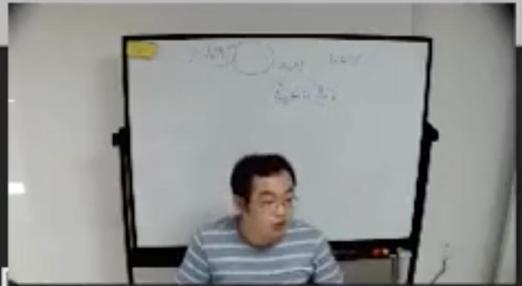
# 堆： 优先队列

普通队列	(最大/最小) 堆
尾部入队	尾部可以插入
头部出队	头部可以弹出
先进先出	每次出队权值 (最大/最小的元素)
数组实现	数组实现, <u>逻辑上看成一个堆</u>

# 堆： 优先队列

普通队列	优先队列
尾部入队	尾部可以插入
头部出队	头部可以弹出
先进先出	每次出队权值（最大/最小的元素）
数组实现	数组实现， <u>逻辑上看成一个堆</u>

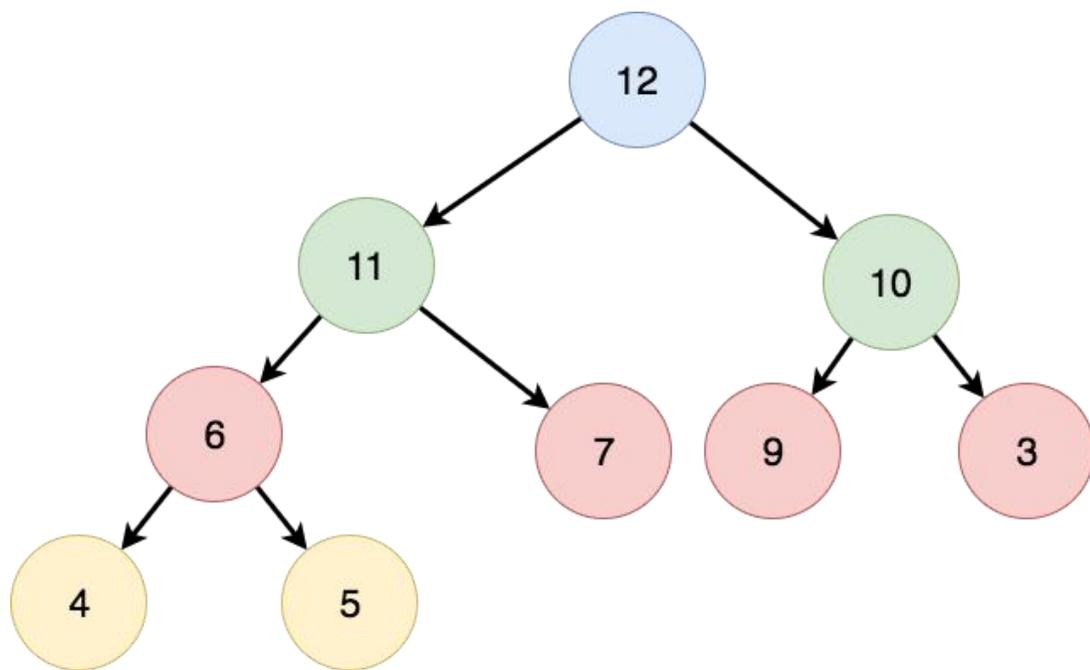
```
1. vim
vim #1 bash #2 bash #3
39 }
40
41 Node *insert_maintain(Node *root) {
42     if (!hasRedChild(root)) return root;
43     if (root->lchild->color == RED && root->rchild->color == RED, {
44         if (!hasRedChild(root->lchild) && !hasRedChild(root->rchild)) return root;
45         root->color = RED;
46         root->lchild->color = root->rchild->color = BLACK;
47         return root;
48     }
49     if (root->lchild->color == RED) {
50         if (!hasRedChild(root->lchild)) return root;
51
52     } else {
53         if (!hasRedChild(root->rchild)) return root;
54
55     }
56 }
57
58
59
60
61 Node *__insert(Node *root, int key) {
62     if (root == NIL) return getNewNode(key);
```



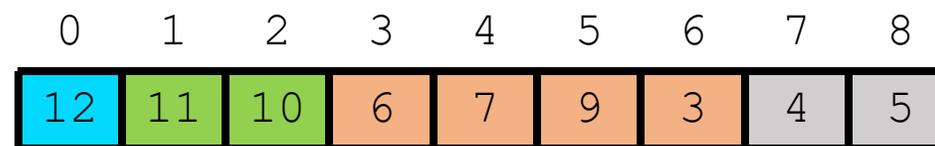
## 优先队列：代码演示

## 二. 堆排序与线性建堆法

# 堆排序



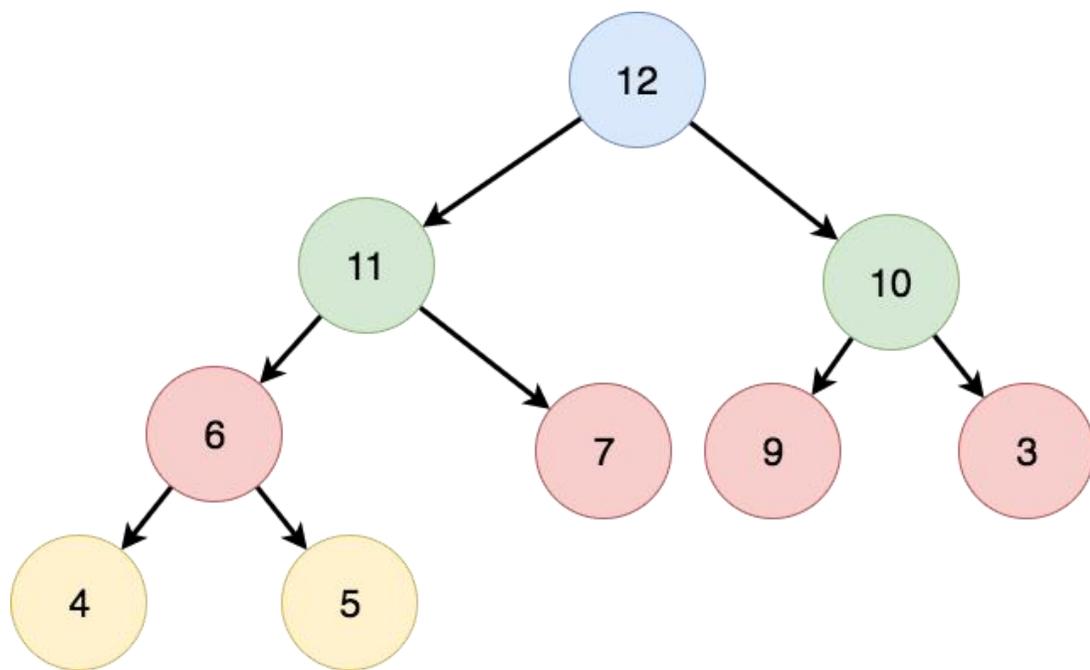
大顶堆



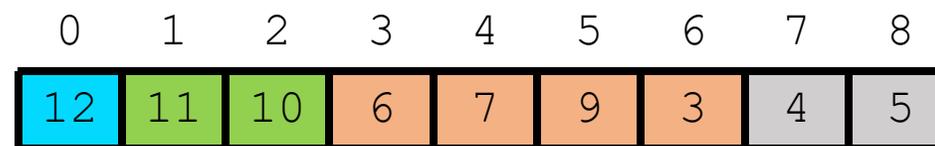
口诀:

- 1、将堆顶元素与堆尾元素交换
- 2、将此操作看做是堆顶元素弹出操作
- 3、按照头部弹出以后的策略调整堆

# 堆排序



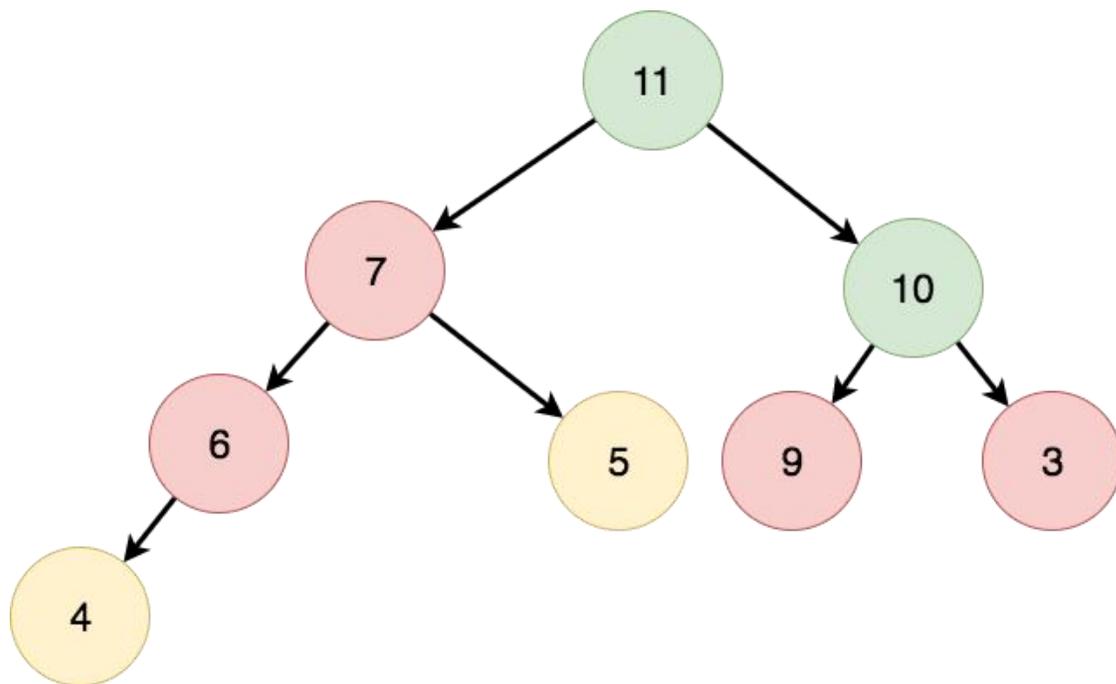
大顶堆



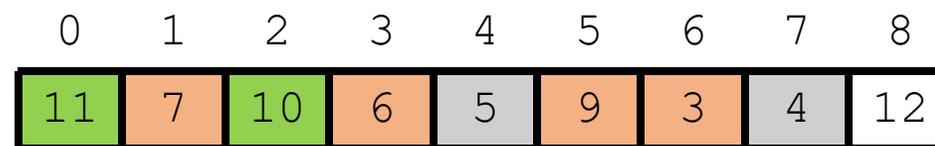
练习题:

- 1、请画出弹出一次以后的堆以及数组
- 2、请画出弹出三次以后的堆以及数组

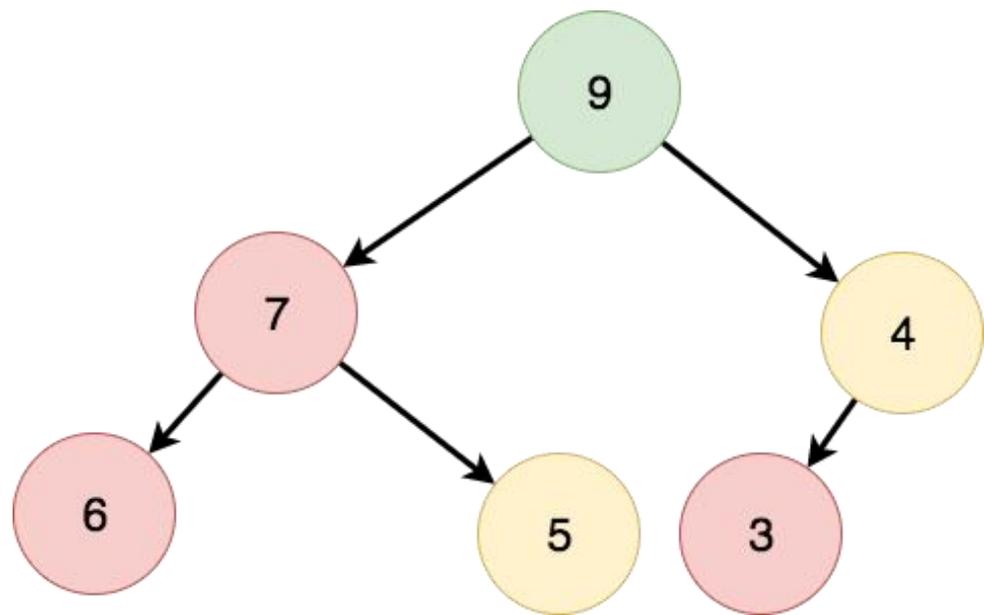
# 堆排序：弹一次



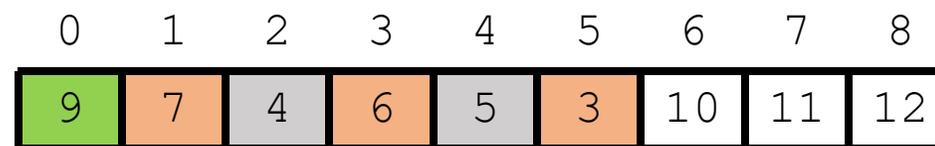
大顶堆



# 堆排序：弹三次



大顶堆

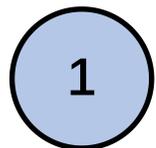


# 普通建堆：向上调整

0	1	2	3	4	5	6
1	2	3	4	5	6	7

大顶堆

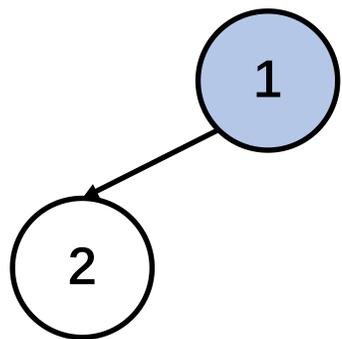
# 普通建堆：向上调整



0	1	2	3	4	5	6
1	2	3	4	5	6	7
0						

大顶堆

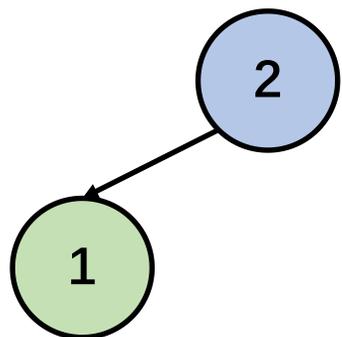
# 普通建堆： 向上调整



0	1	2	3	4	5	6
1	2	3	4	5	6	7
0						

大顶堆

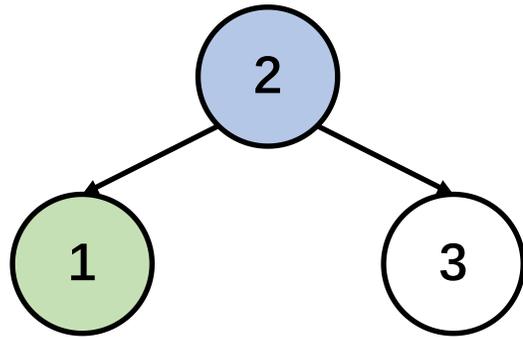
# 普通建堆： 向上调整



0	1	2	3	4	5	6
2	1	3	4	5	6	7
0	1					

大顶堆

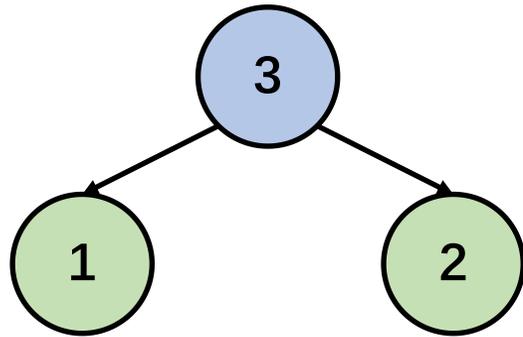
# 普通建堆： 向上调整



0	1	2	3	4	5	6
2	1	3	4	5	6	7
0	1					

大顶堆

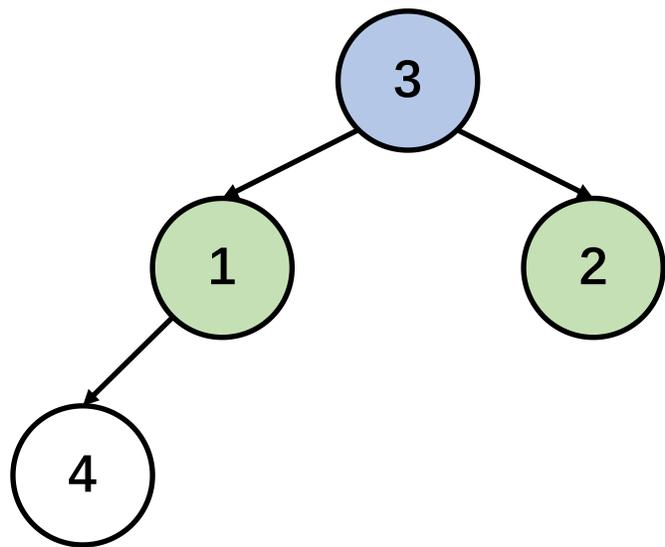
# 普通建堆：向上调整



0	1	2	3	4	5	6
3	1	2	4	5	6	7
0	1	1				

大顶堆

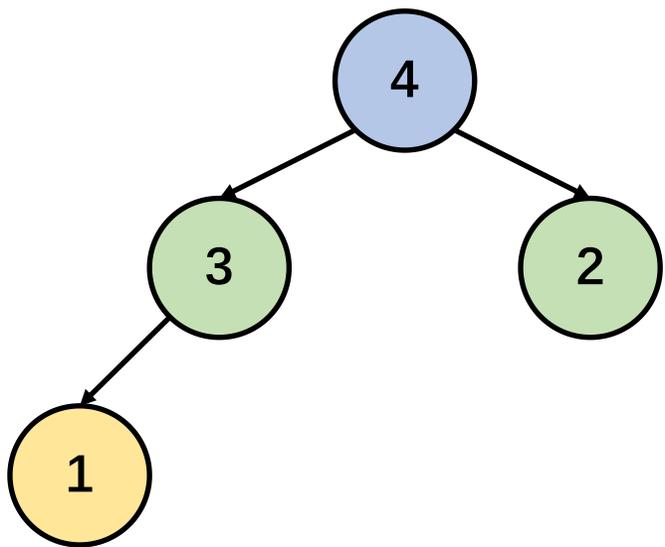
# 普通建堆：向上调整



大顶堆

0	1	2	3	4	5	6
3	1	2	4	5	6	7
0	1	1				

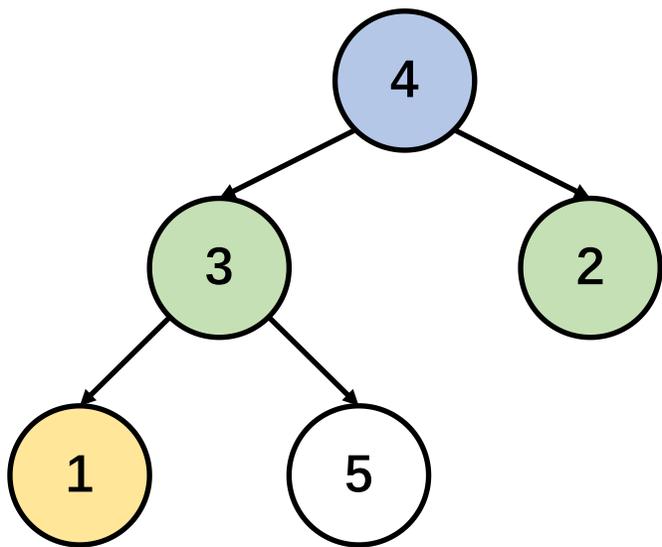
# 普通建堆：向上调整



大顶堆

0	1	2	3	4	5	6
4	3	2	1	5	6	7
0	1	1	2			

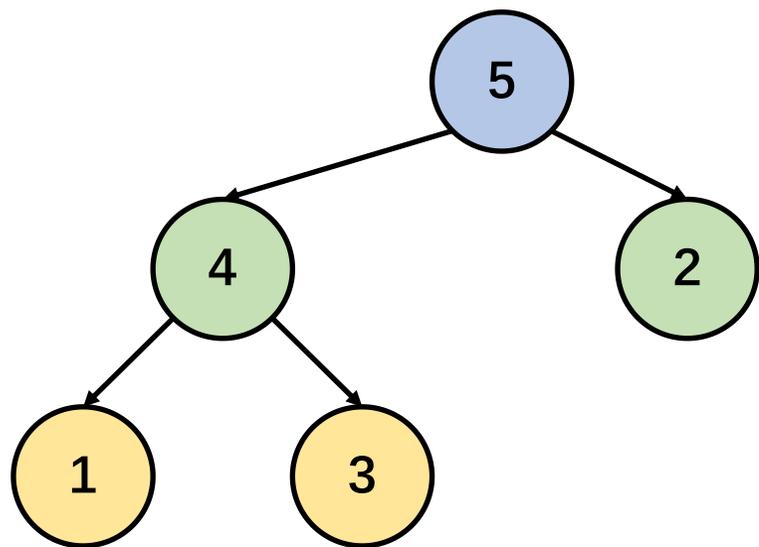
# 普通建堆：向上调整



大顶堆

0	1	2	3	4	5	6
4	3	2	1	5	6	7
0	1	1	2			

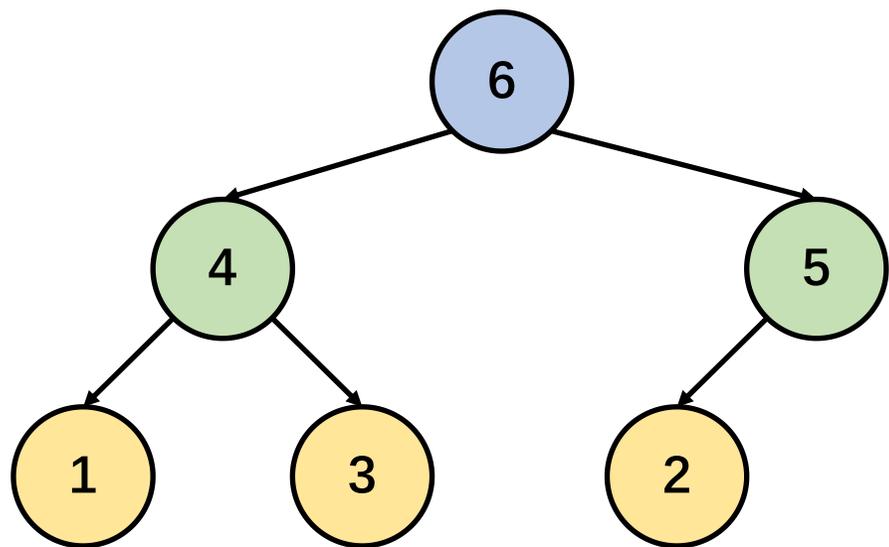
# 普通建堆：向上调整



大顶堆

0	1	2	3	4	5	6
5	4	2	1	3	6	7
0	1	1	2	2		

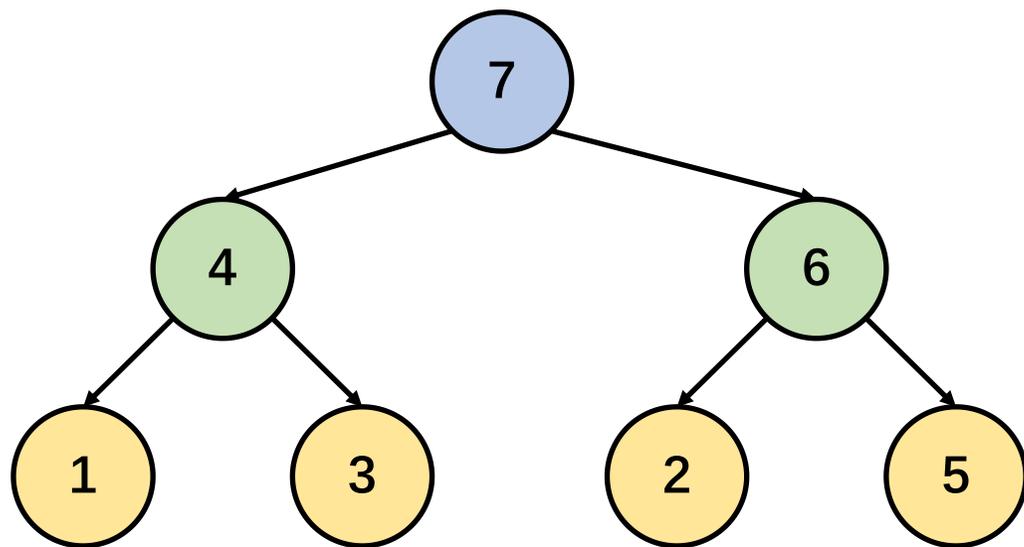
# 普通建堆：向上调整



大顶堆

0	1	2	3	4	5	6
6	4	5	1	3	2	7
0	1	1	2	2	2	

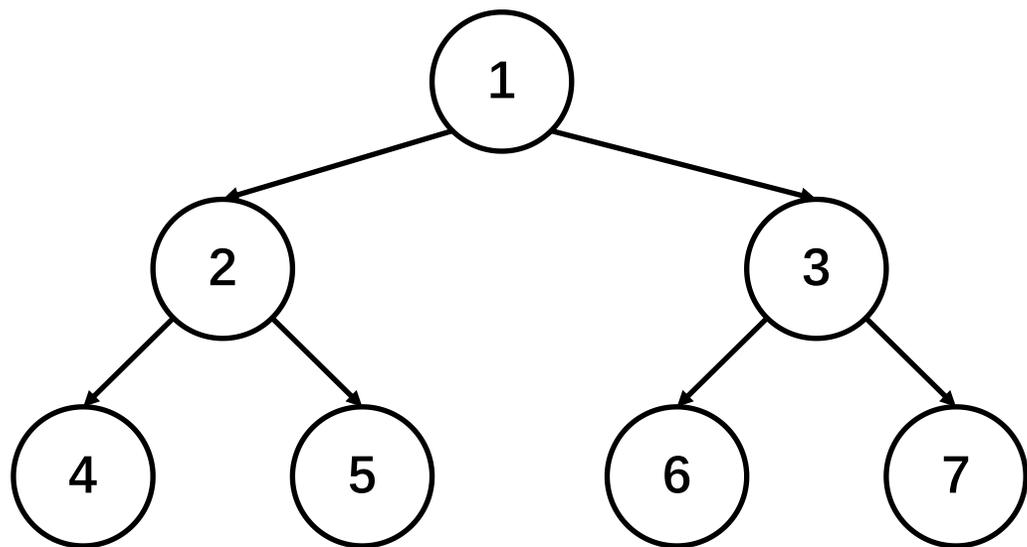
# 普通建堆：向上调整



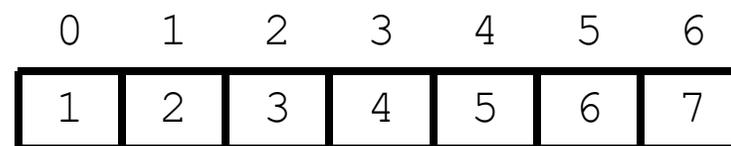
大顶堆

0	1	2	3	4	5	6
7	4	6	1	3	2	5
0	1	1	2	2	2	2

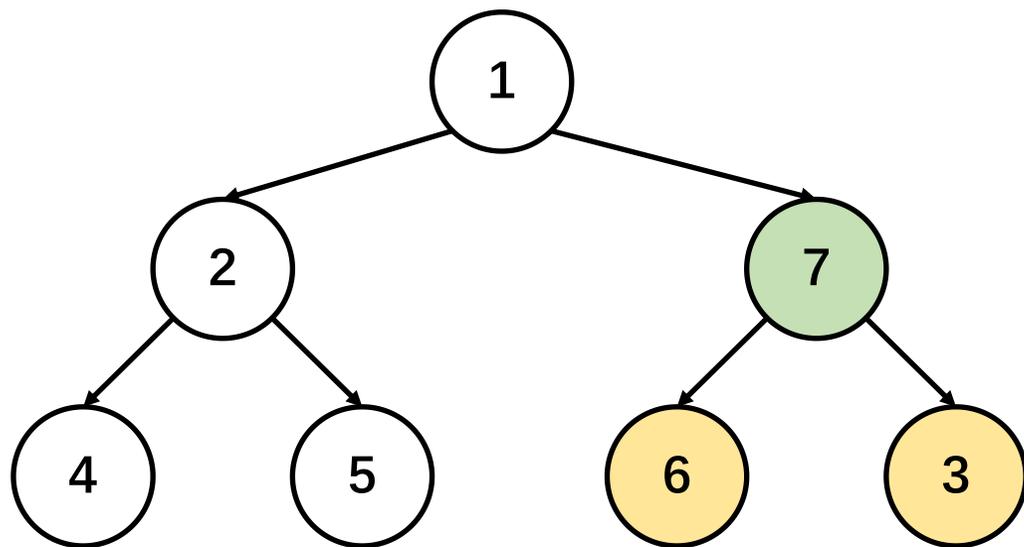
# 线性建堆：向下调整



大顶堆



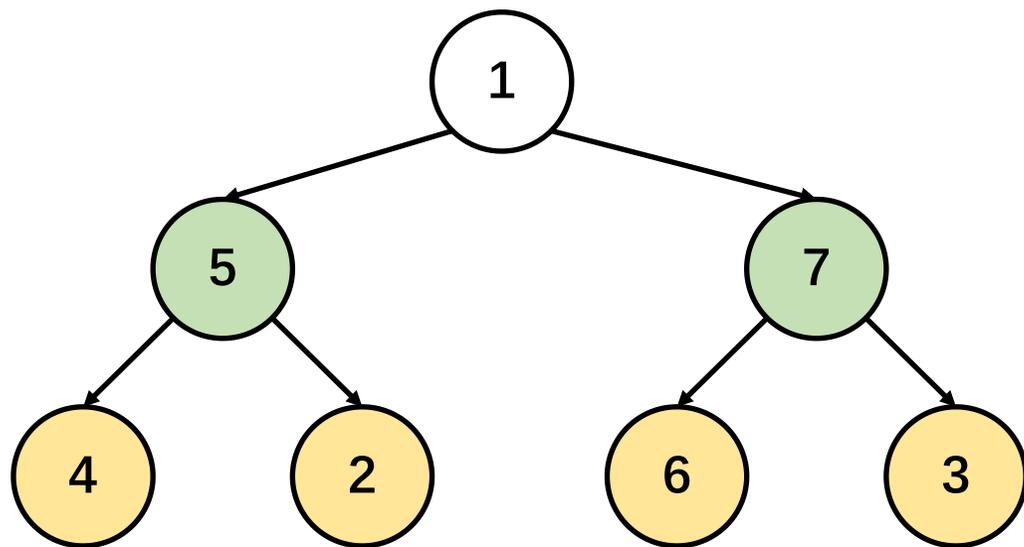
# 线性建堆：向下调整



大顶堆

0	1	2	3	4	5	6
1	2	7	4	5	6	3
		1			0	0

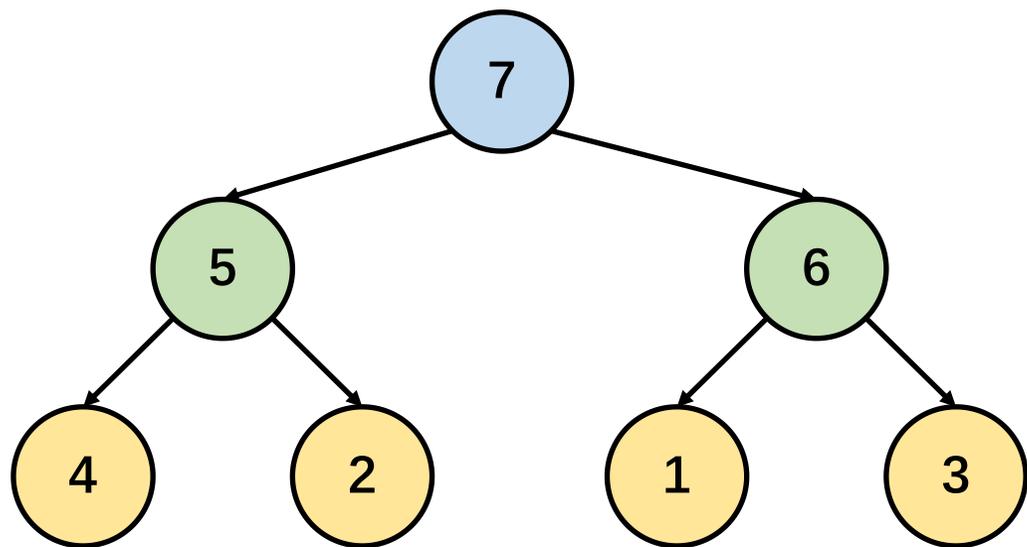
# 线性建堆：向下调整



大顶堆

0	1	2	3	4	5	6
1	5	7	4	2	6	3
	1	1	0	0	0	0

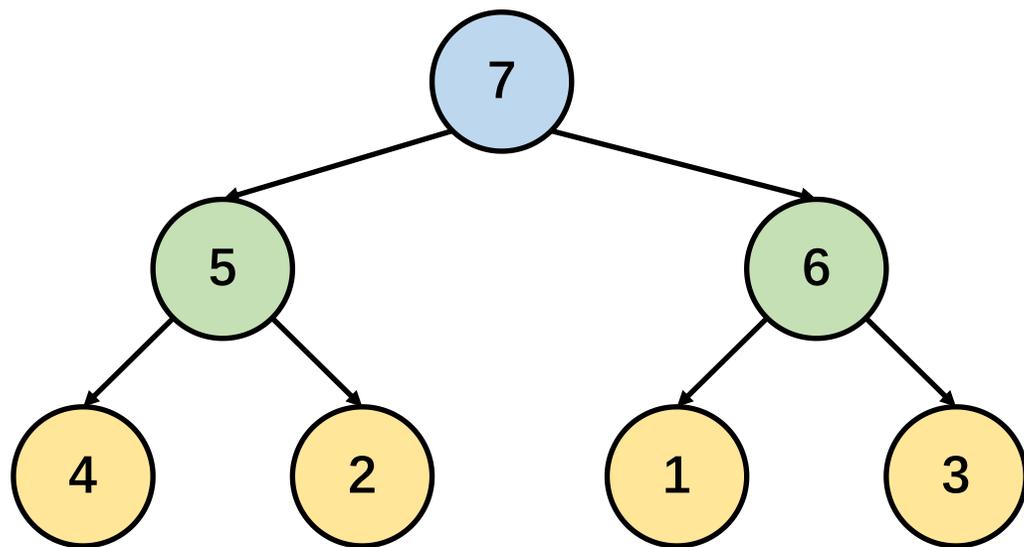
# 线性建堆：向下调整



大顶堆

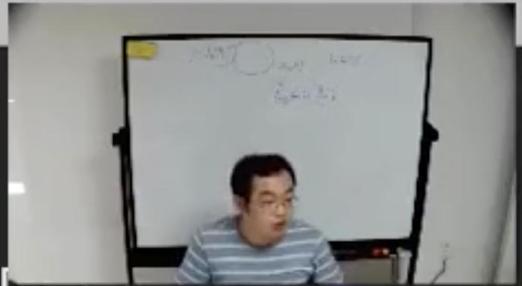
0	1	2	3	4	5	6
7	5	6	4	2	1	3
2	1	1	0	0	0	0

# 线性建堆法复杂度推导



大顶堆

```
1. vim
vim #1 bash #2 bash #3
39 }
40
41 Node *insert_maintain(Node *root) {
42     if (!hasRedChild(root)) return root;
43     if (root->lchild->color == RED && root->rchild->color == RED, {
44         if (!hasRedChild(root->lchild) && !hasRedChild(root->rchild)) return root;
45         root->color = RED;
46         root->lchild->color = root->rchild->color = BLACK;
47         return root;
48     }
49     if (root->lchild->color == RED) {
50         if (!hasRedChild(root->lchild)) return root;
51
52     } else {
53         if (!hasRedChild(root->rchild)) return root;
54
55     }
56 }
57
58
59
60
61 Node *__insert(Node *root, int key) {
62     if (root == NIL) return getNewNode(key);
```



## 堆排序：代码演示

## 三. 优化：哈夫曼编码

# 哈夫曼编码

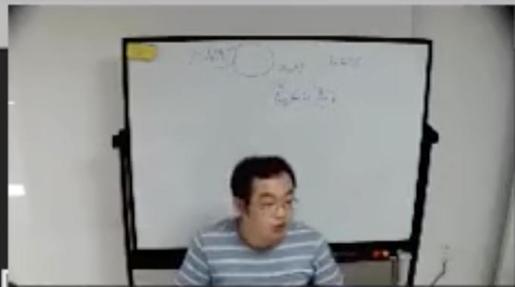
哈夫曼编码生成过程：

1. 首先，统计得到每一种字符的概率
2. 每次将最低频率的两个节点合并成一棵子树
3. 经过了  $n-1$  轮合并，就得到了一棵哈夫曼树
4. 按照左0，右1的形式，将编码读取出来

$(a, 0.5), (b, 0.2)$

$(c, 0.1), (d, 0.2)$

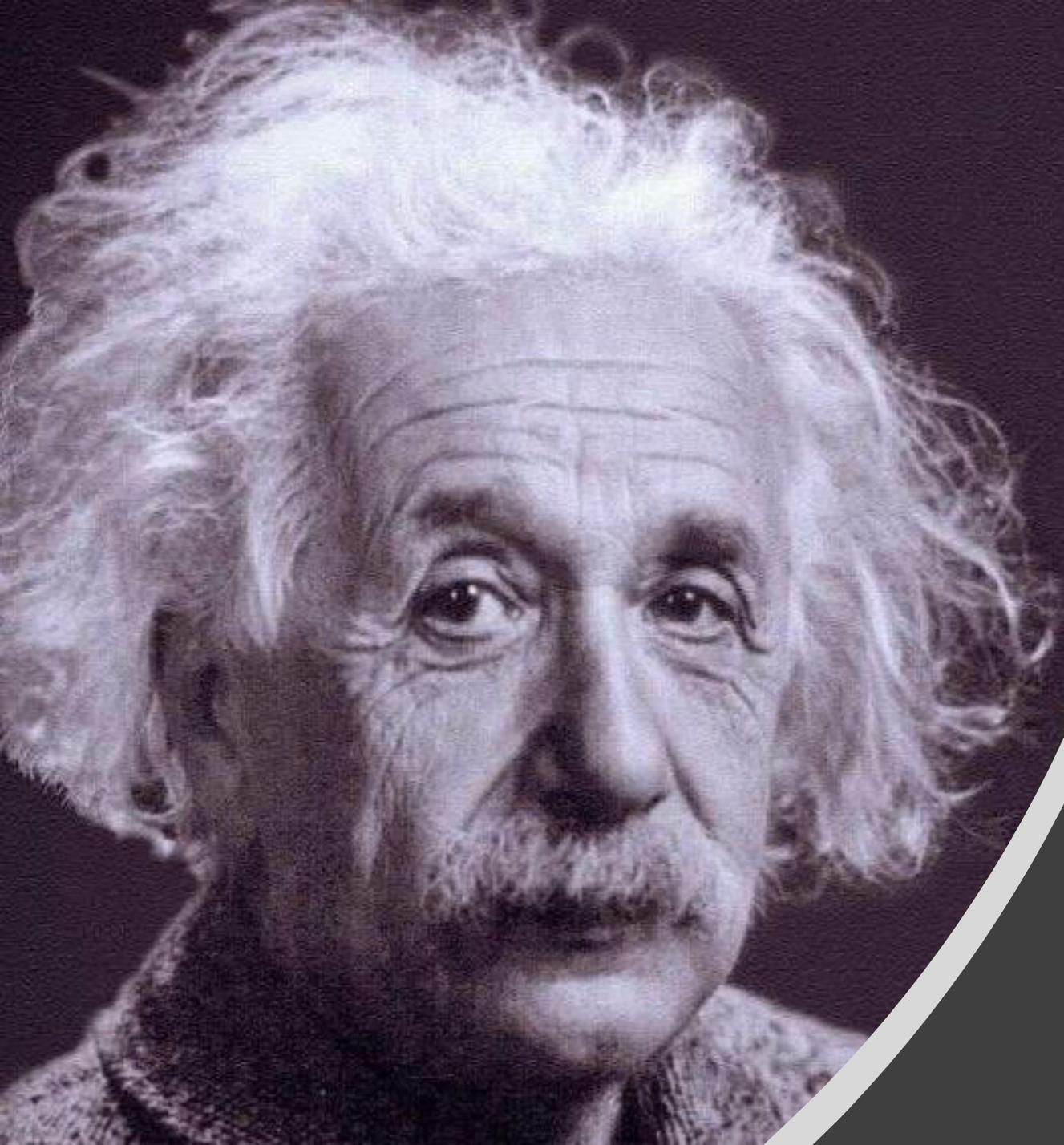
```
vim #1 bash #2 bash #3
39 }
40
41 Node *insert_maintain(Node *root) {
42     if (!hasRedChild(root)) return root;
43     if (root->lchild->color == RED && root->rchild->color == RED, {
44         if (!hasRedChild(root->lchild) && !hasRedChild(root->rchild)) return root;
45         root->color = RED;
46         root->lchild->color = root->rchild->color = BLACK;
47         return root;
48     }
49     if (root->lchild->color == RED) {
50         if (!hasRedChild(root->lchild)) return root;
51
52
53     } else {
54         if (!hasRedChild(root->rchild)) return root;
55
56     }
57
58 }
```



## 哈夫曼编码：代码优化

```
61 Node *__insert(Node *root, int key) {
```

```
62     if (root == NIL) return getNewNode(key);
```



为什么  
会出一样的题目？