平衡树基础

胡船长

初航我带你，远航靠自己

本章题目

本章最大的练习题，就是：

学会每一个结构的代码细节

本期内容

一. （2）二叉排序树

二. （2）AVL 树

三. （4）红黑树

四. （4）B-树

一. 二叉排序树

二叉排序树

名称：二叉排序树、二叉搜索树

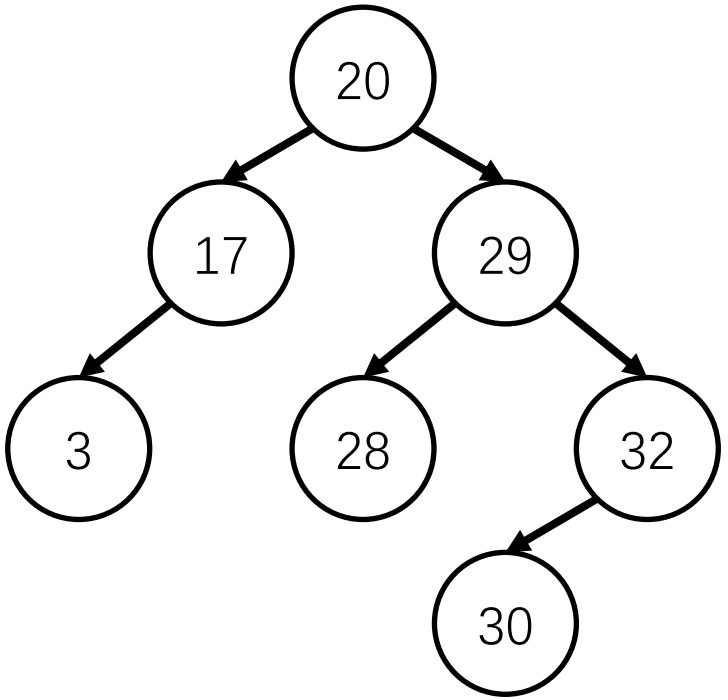
性质：

1、左子树 < 根节点

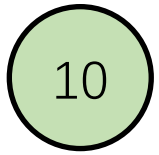
2、右子树 > 根节点

用途：

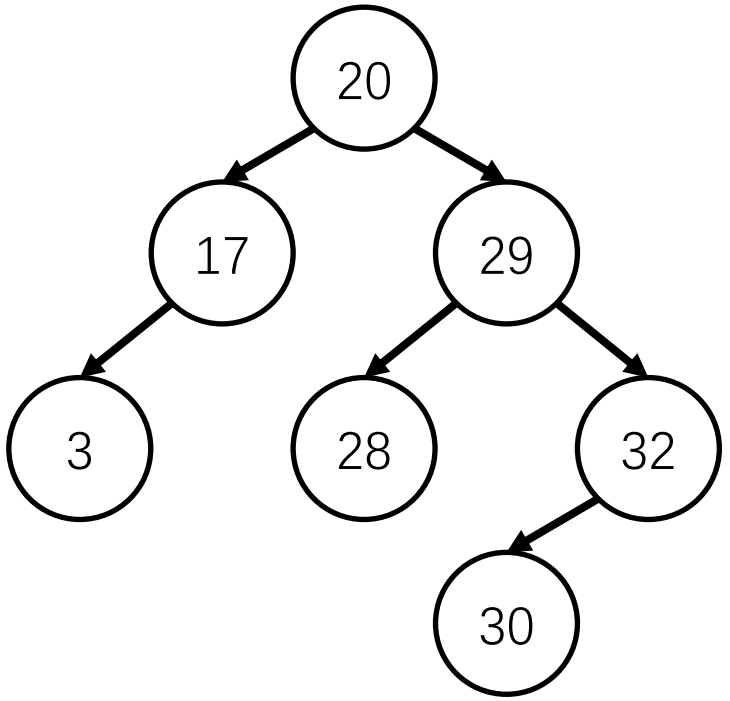
解决与排名相关的检索需求



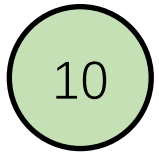
二叉排序树的插入



待插入：10

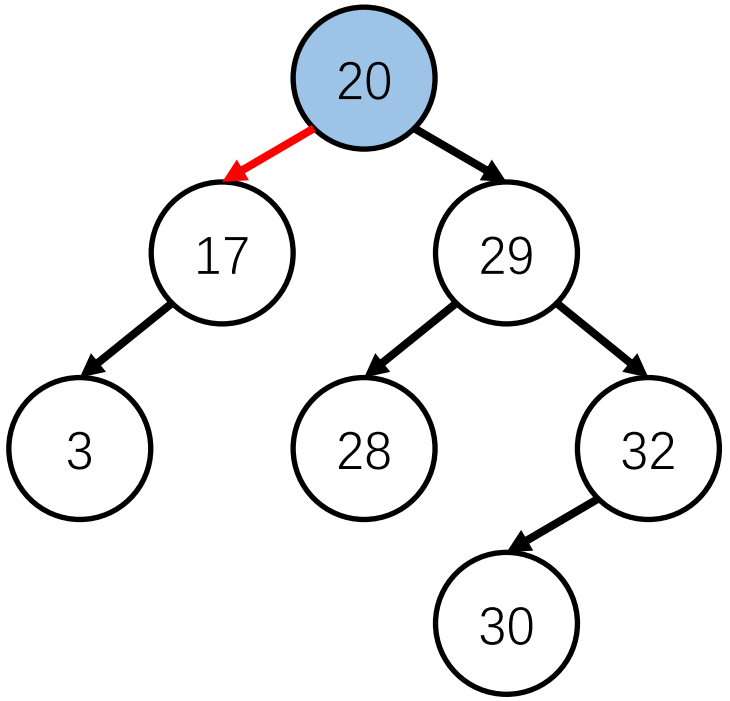


二叉排序树的插入

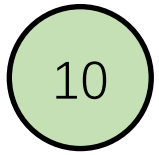


10 < 20

插入到左子树

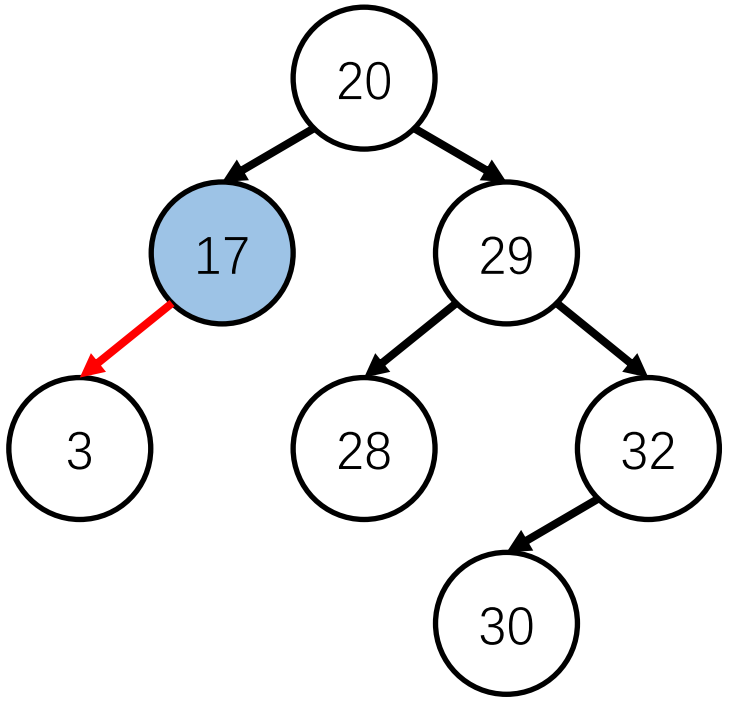


二叉排序树的插入

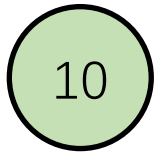


10 < 17

插入到左子树

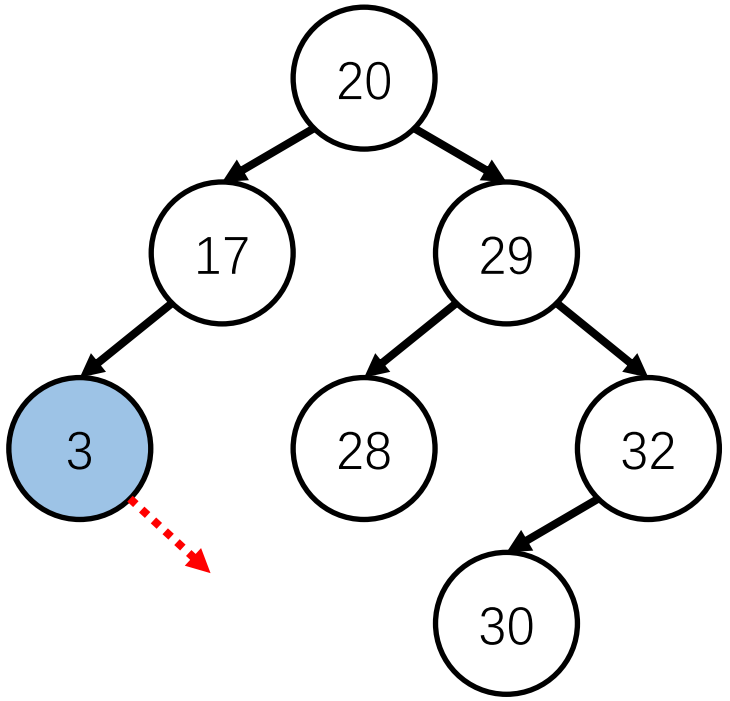


二叉排序树的插入



10 > 3

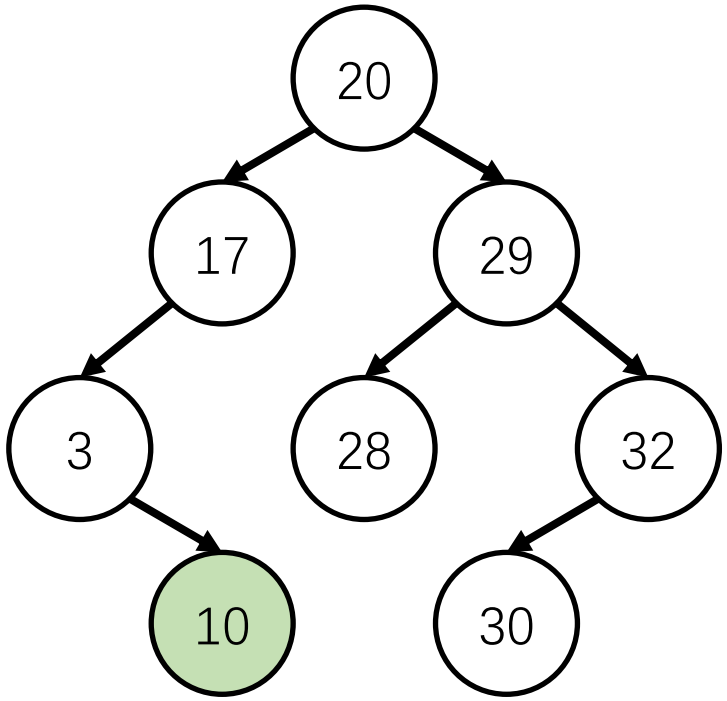
插入到右子树



二叉排序树的插入

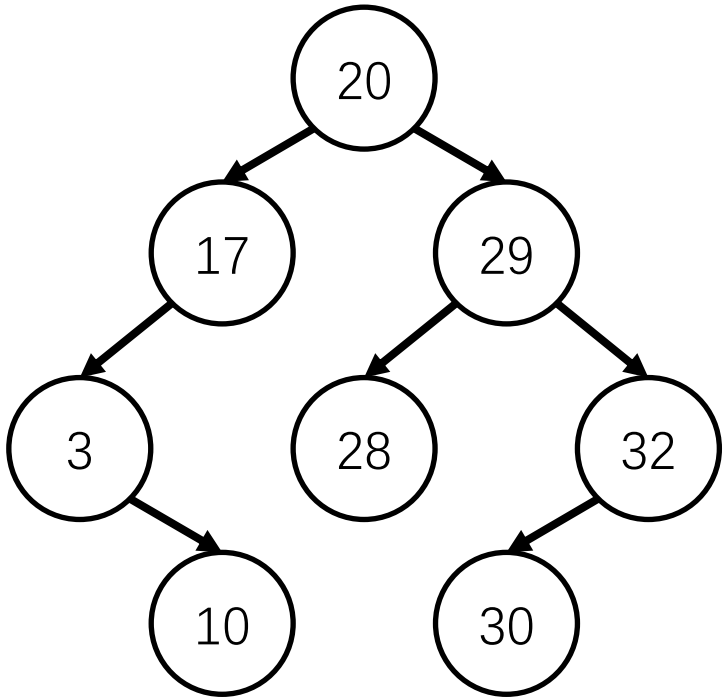
10 > 3

插入到右子树



二叉排序树的删除

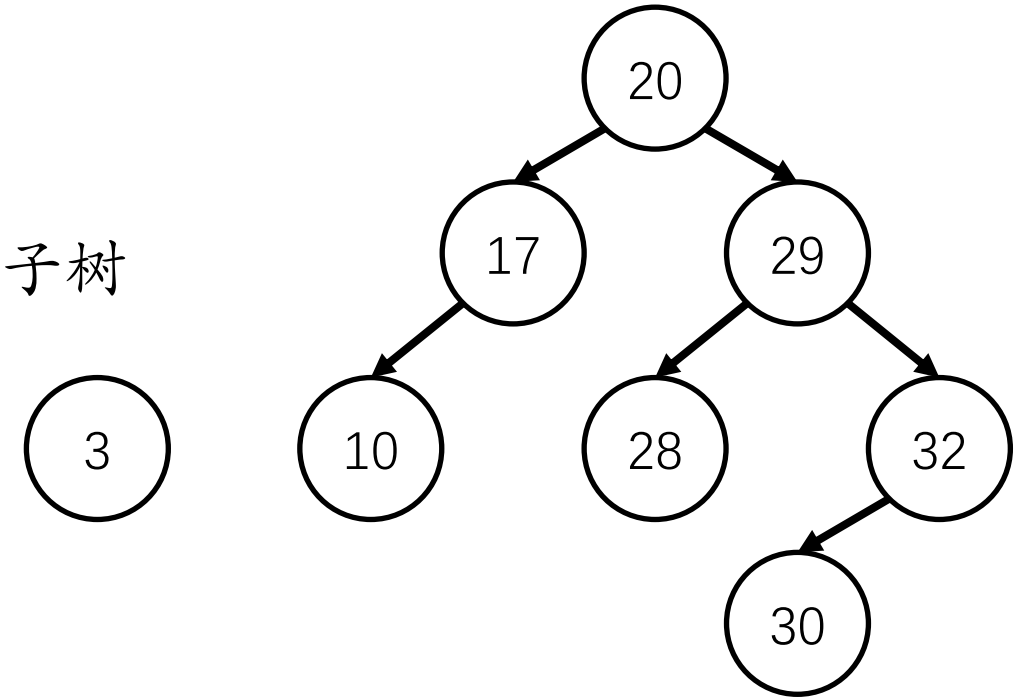
1、删除叶子节点
2、删除出度为1的节点
3、删除出度为2的节点



二叉排序树的删除

删除出度为1的节点

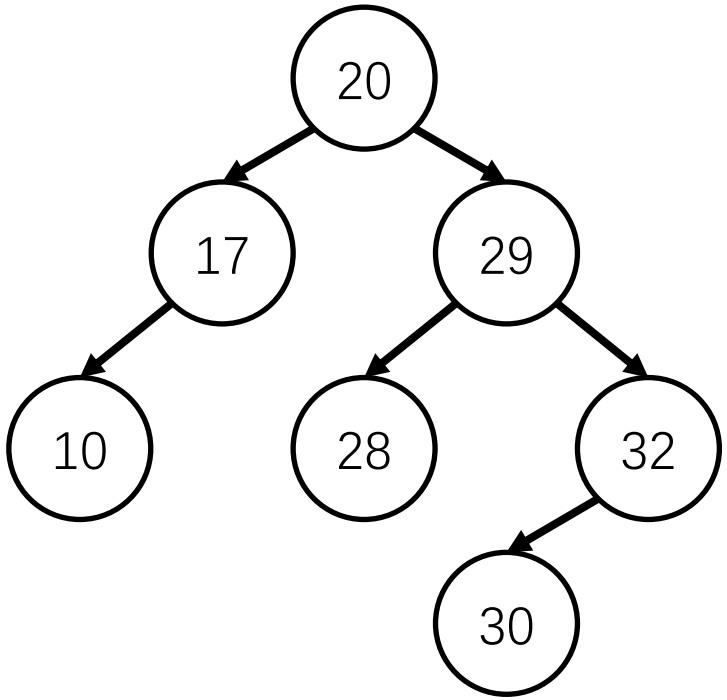
提升3的唯一子树



二叉排序树的删除

删除出度为1的节点

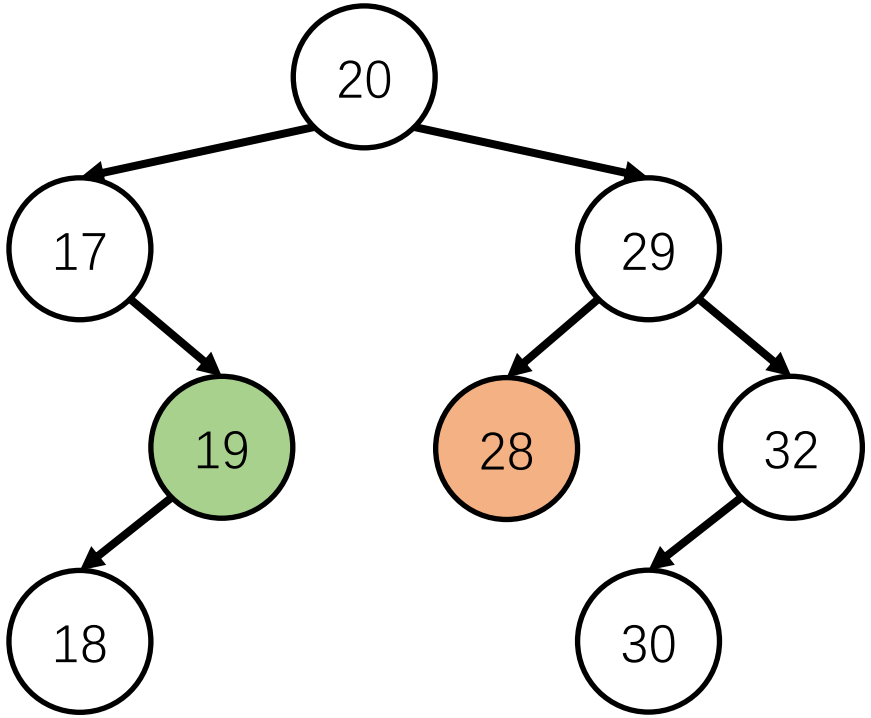
提升3的唯一子树



二叉排序树的删除

删除出度为2的节点

找到前驱或者后继替换后
转换为度为1的节点问题



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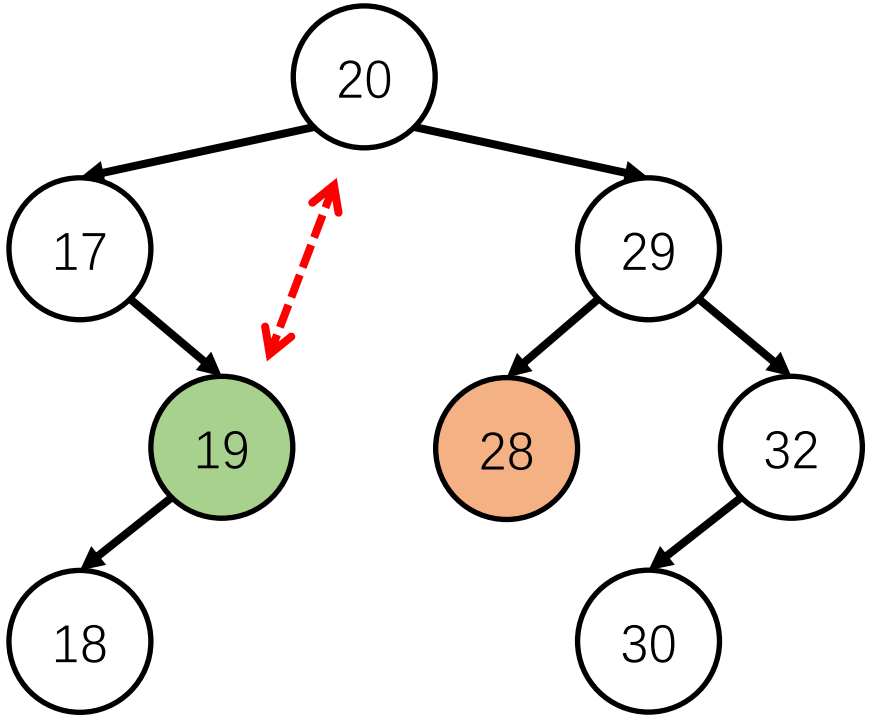

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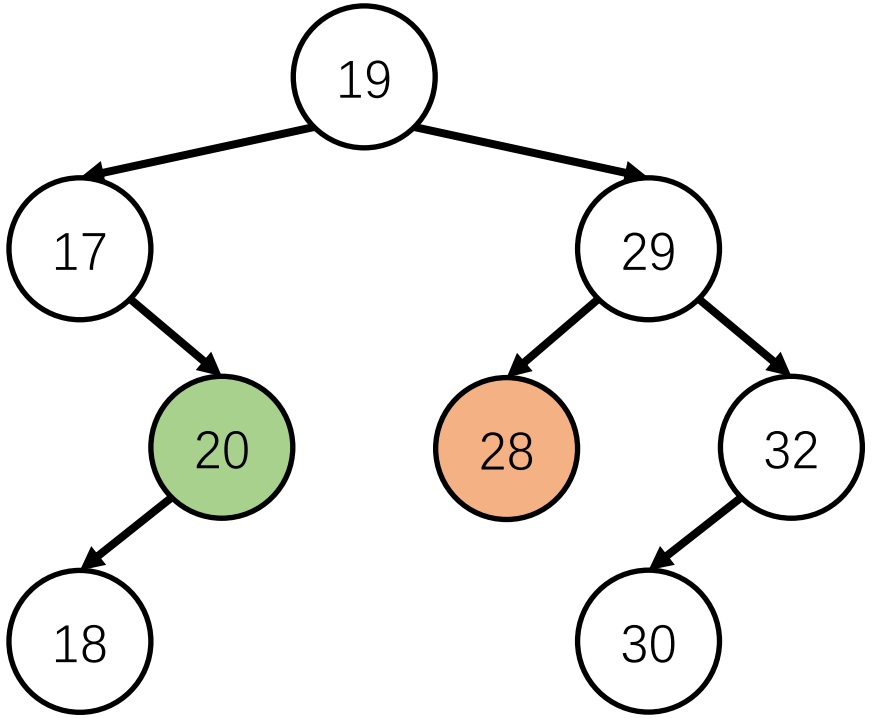

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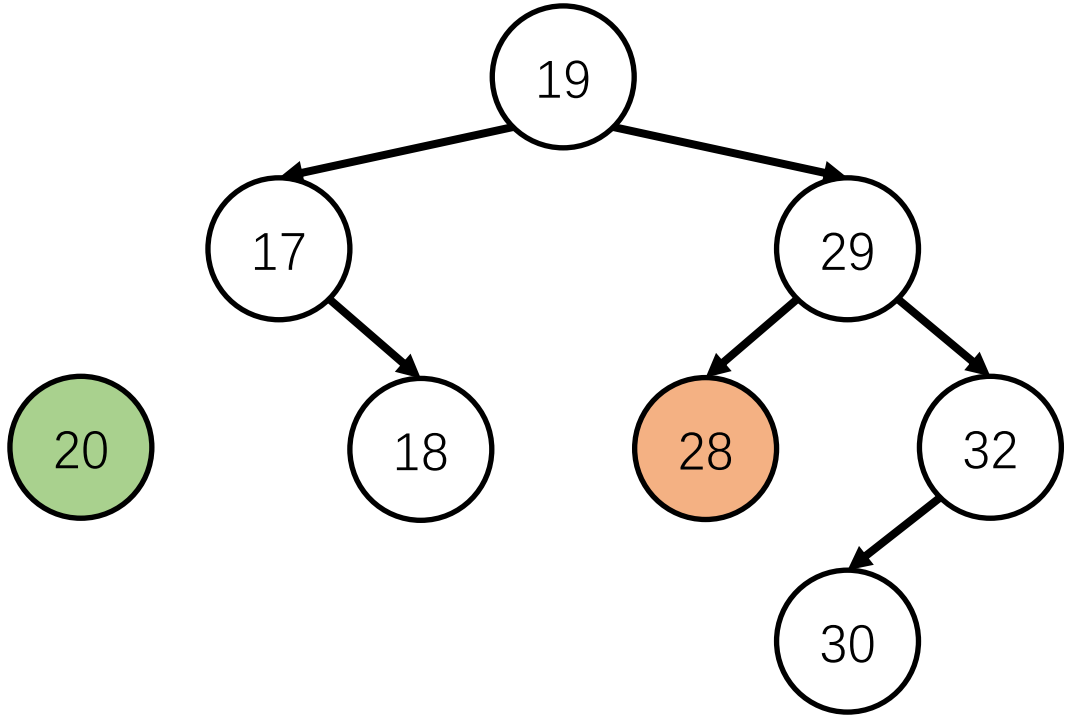

二叉排序树的删除

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找到前驱或者后继替换后
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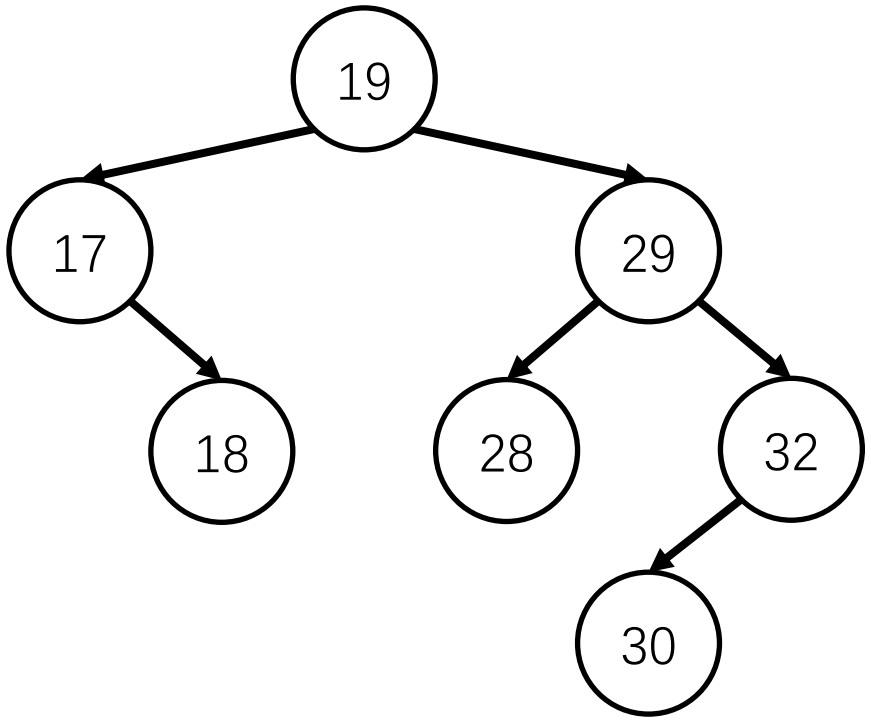
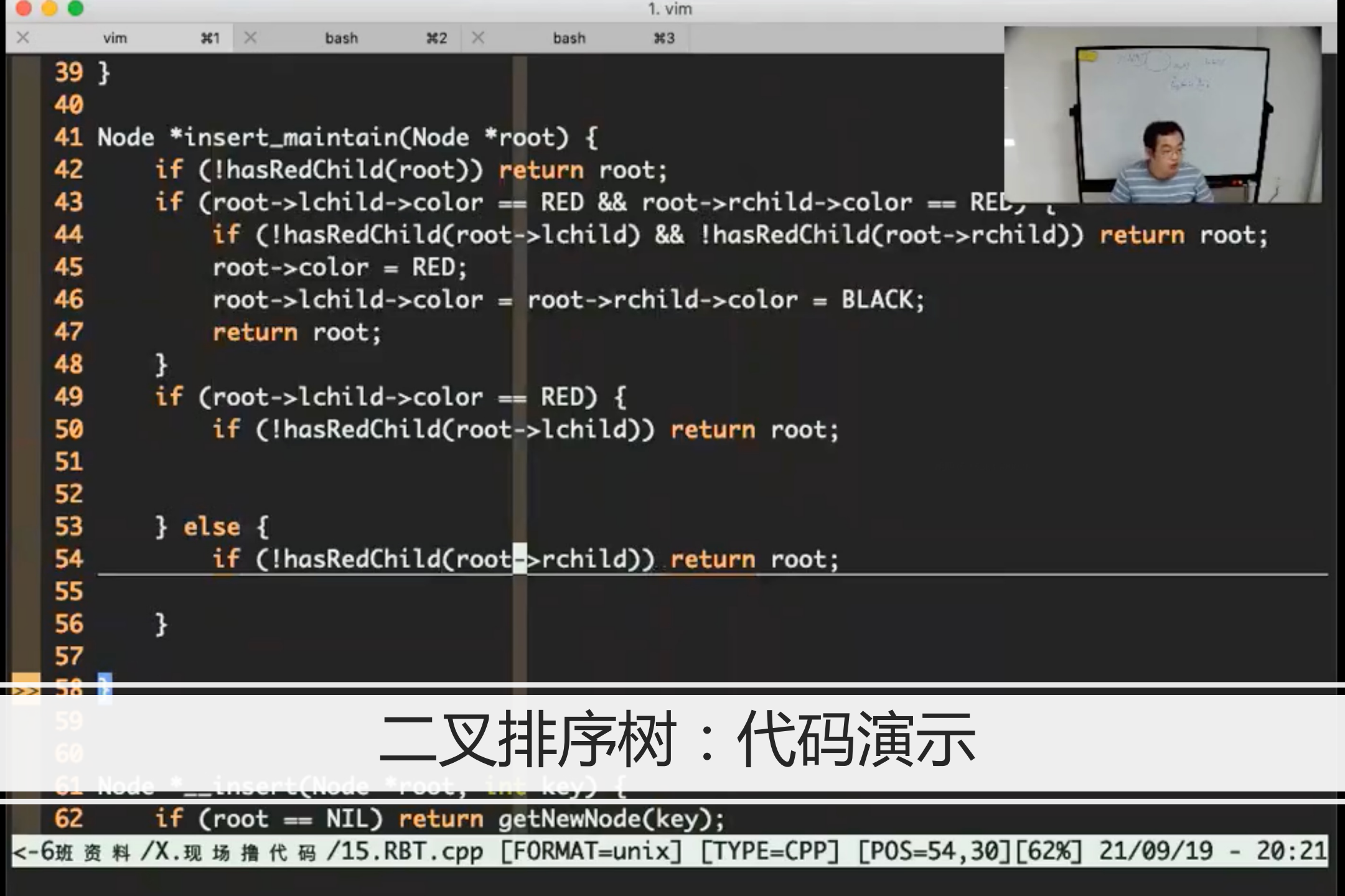



1. vim
vim ⌘1
bash ⌘2
bash ⌘3
39 }
40
41 Node *insert_maintain(Node *root) {
42 if (!hasRedChild(root)) return root;
43 if (root->lchild->color == RED && root->rchild->color == RED) {
44 if (!hasRedChild(root->lchild) && !hasRedChild(root->rchild)) return root;
45 root->color = RED;
46 root->lchild->color = root->rchild->color = BLACK;
47 return root;
48 }
49 if (root->lchild->color == RED) {
50 if (!hasRedChild(root->lchild)) return root;
51
52
53 } else {
54 if (!hasRedChild(root->rchild)) return root;
55
56 }
57
58 }
59
60
61 Node *__insert(Node *root, int key) {
62 if (root == NIL) return getNewNode(key);
<-6班 资 料 /X.现 场 撸 代 码 /15.RBT.cpp [FORMAT=unix] [TYPE=CPP] [POS=54,30][62%] 21/09/19 - 20:21

二叉排序树：代码演示

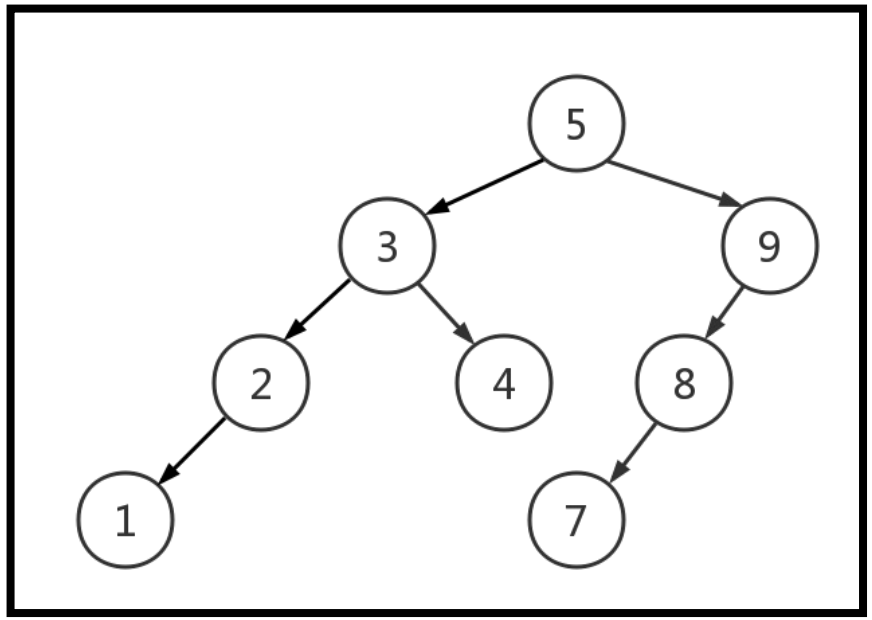
二. AVL 树

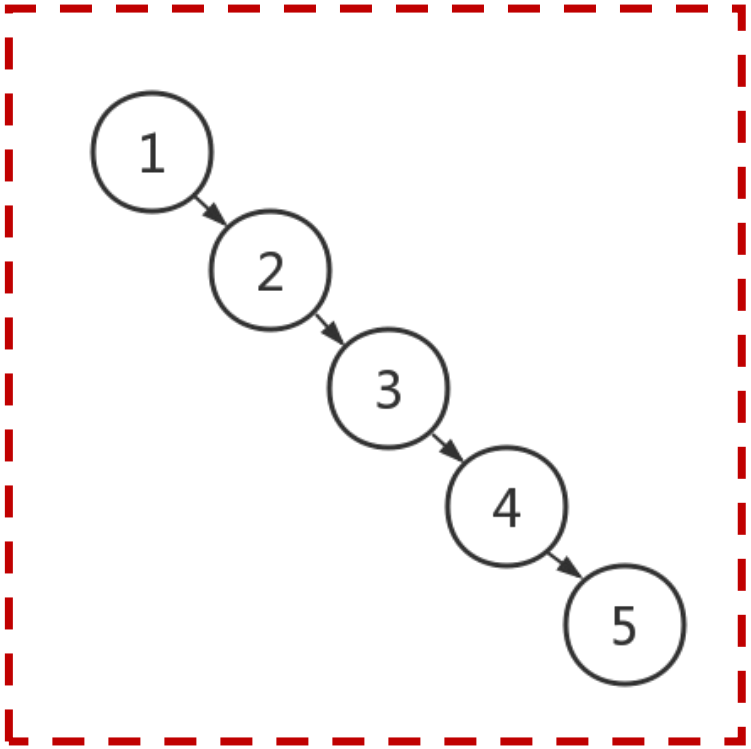
随堂练习-1

请按照如下顺序插入数字，画出对应的二叉搜索树

```
1 : [ 5 9 8 3 2 4 1 7 ]
2 : [ 1 2 3 4 5 ]
```

随堂练习-1





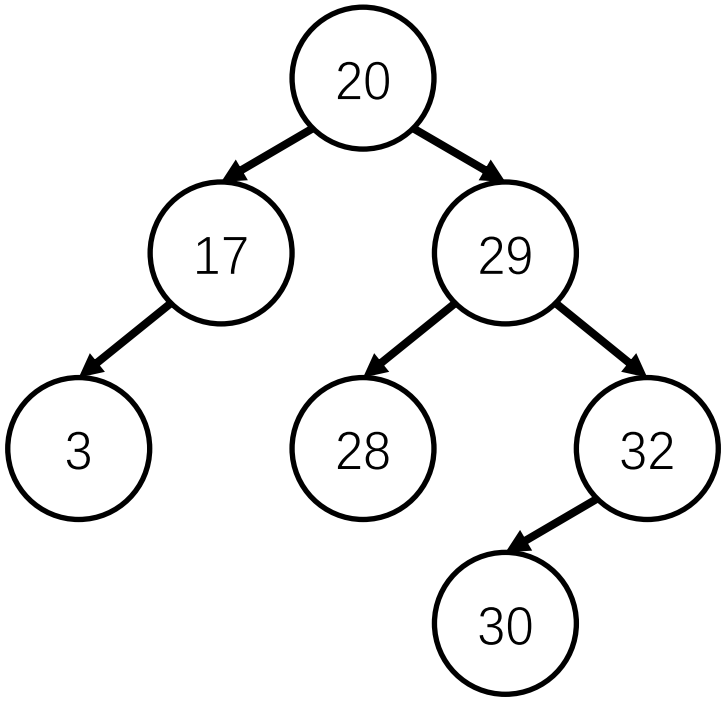
AVL 树

名称：AVL 树
发明者：
G.M. **A**delson-**V**elsky
E.M. **L**andis
年代：1962 年（61 岁）

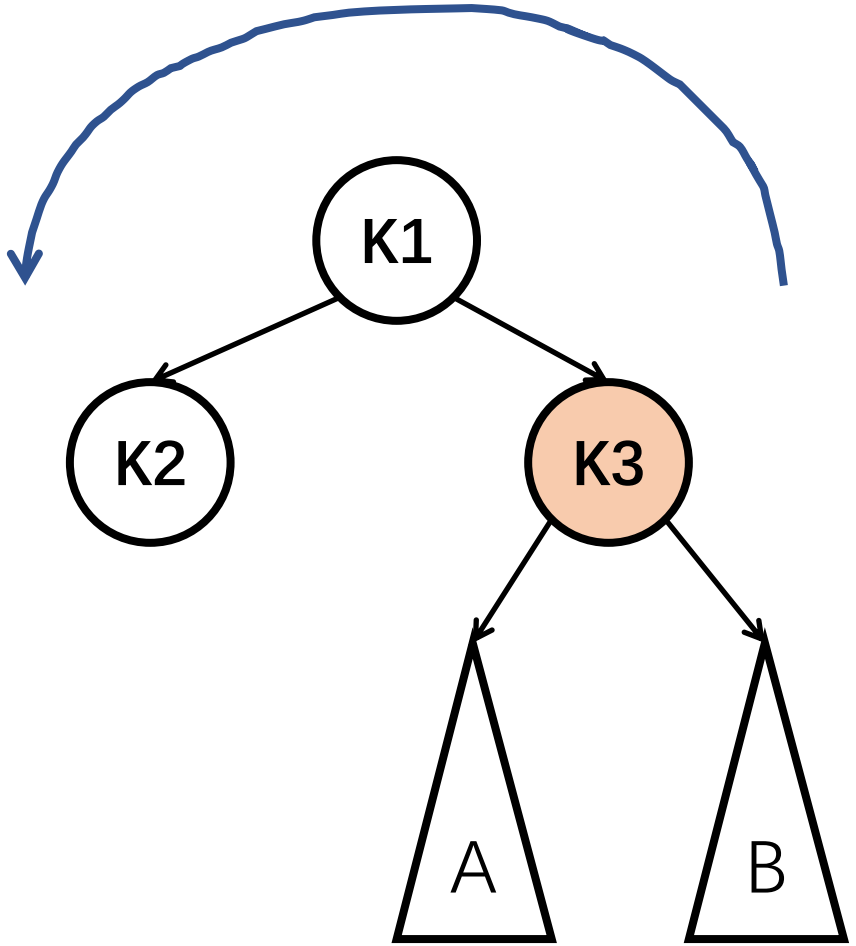
性质：

```
| H(left) - H(right) | <= 1
```

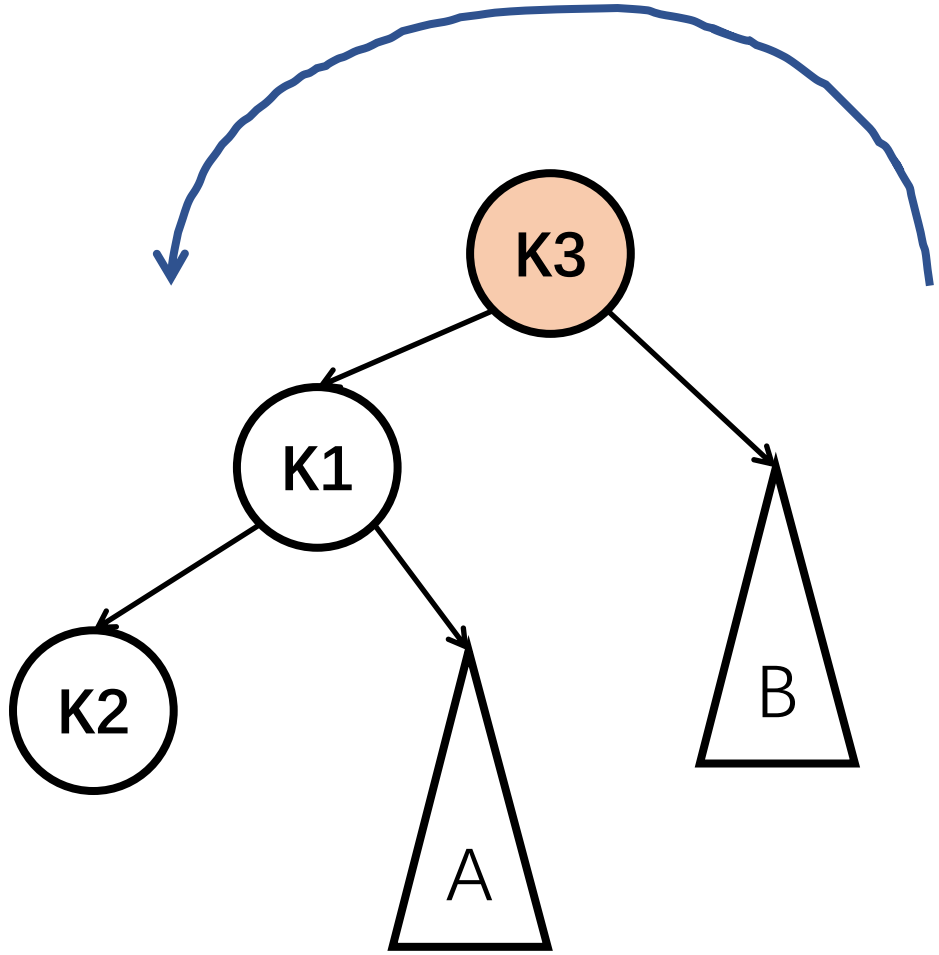
优点：
由于对每个节点的左右子树的树高做了限制，所以整棵树不会退化成一个链表



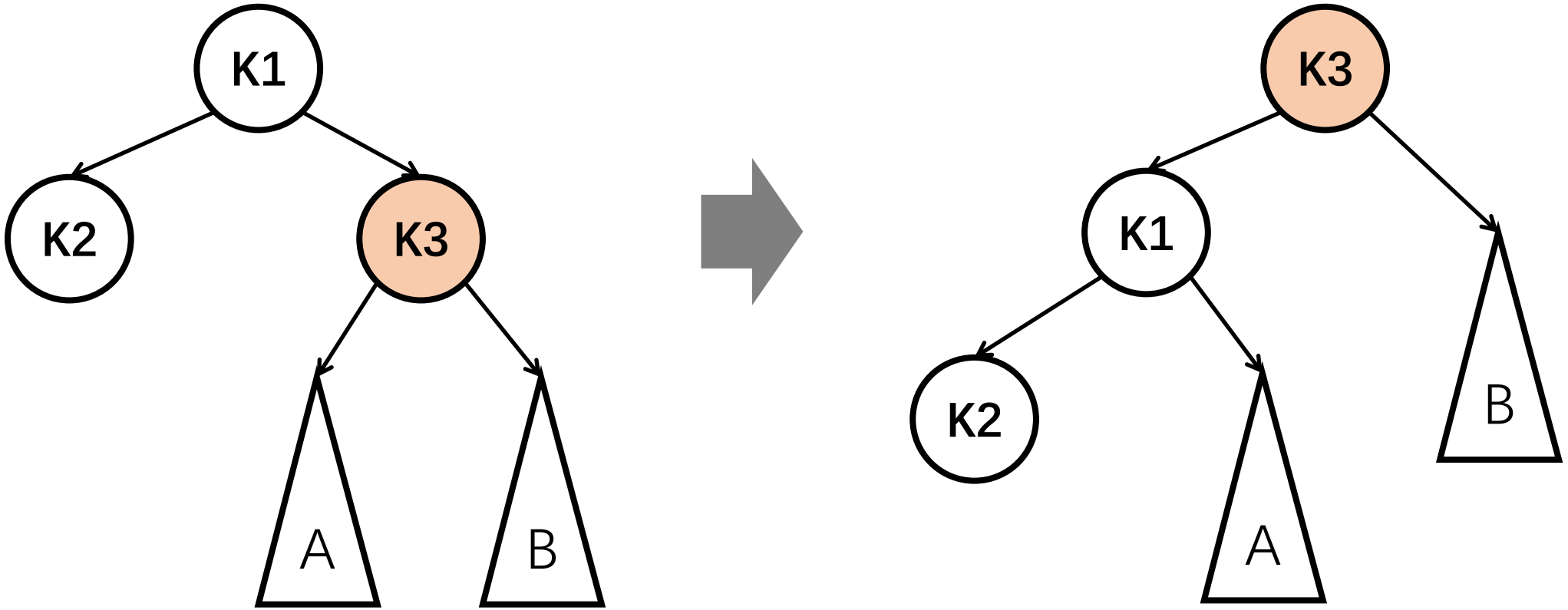
AVL 树-左旋



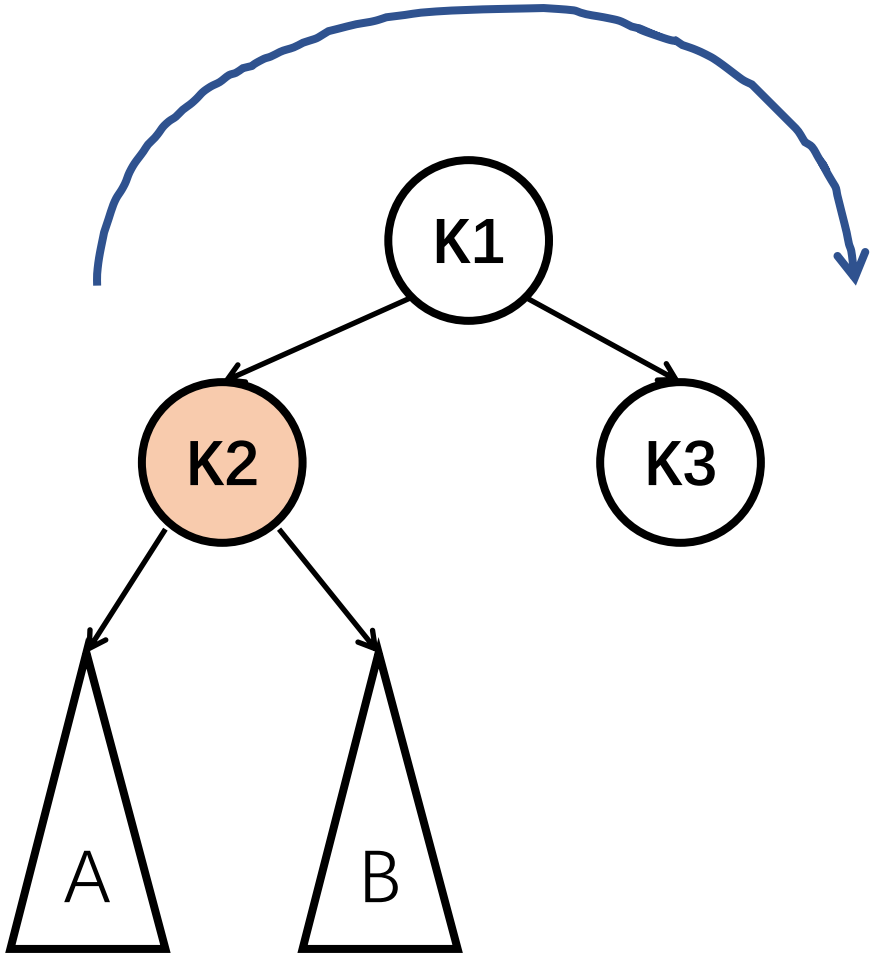
AVL 树-左旋



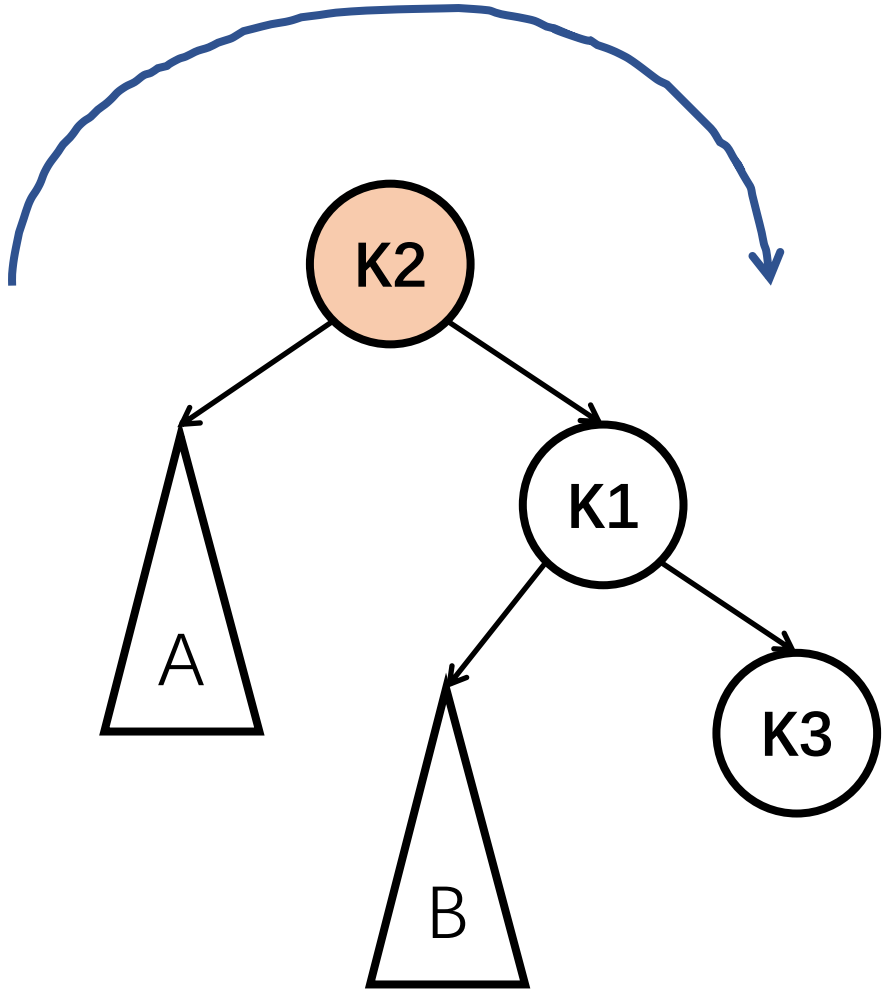
AVL 树-左旋



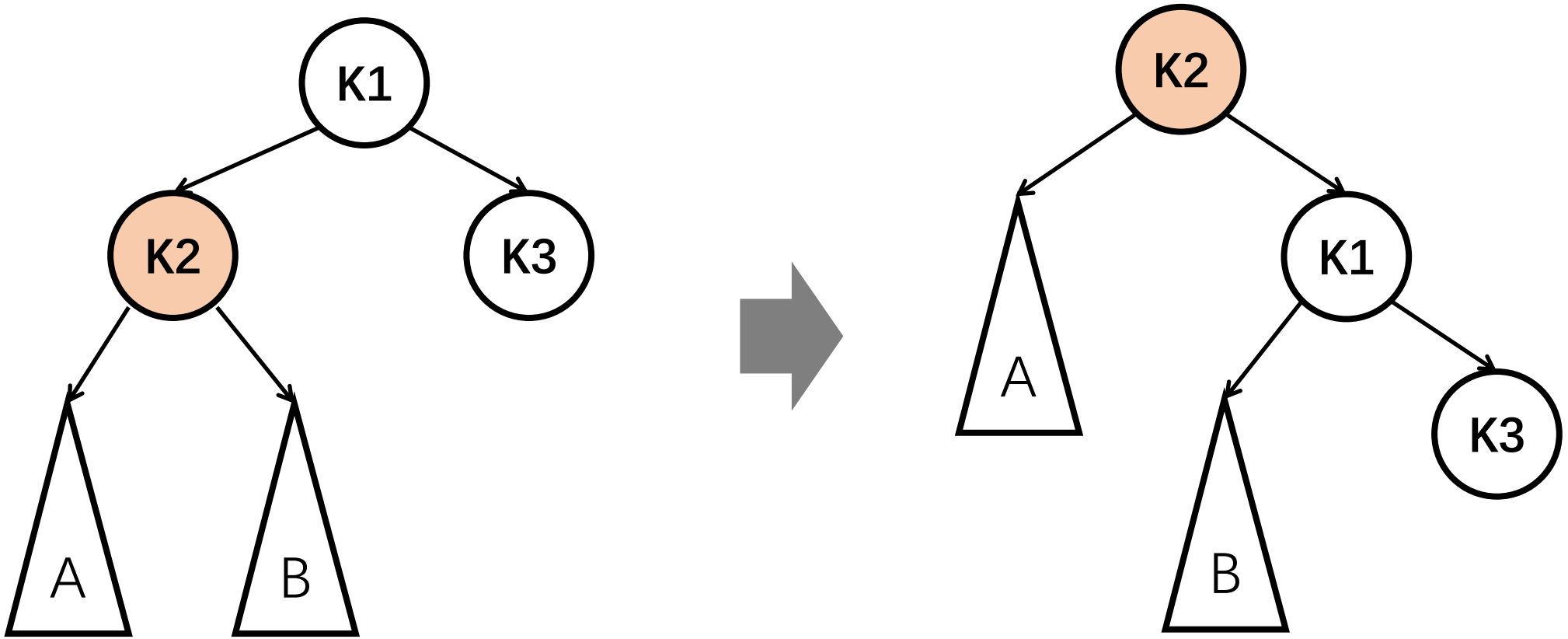
AVL 树-右旋



AVL 树-右旋

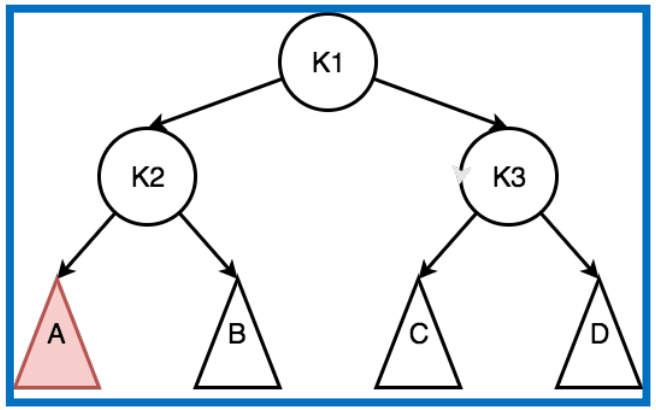


AVL 树-右旋

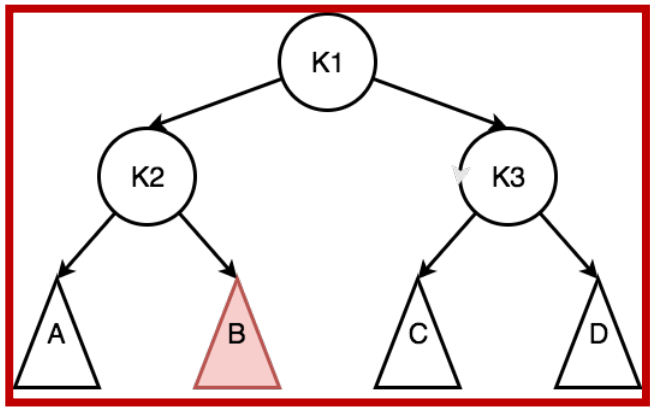


AVL 树-失衡类型

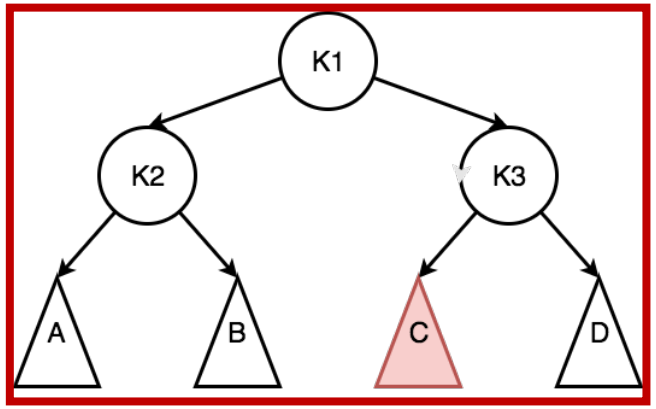
LL 型



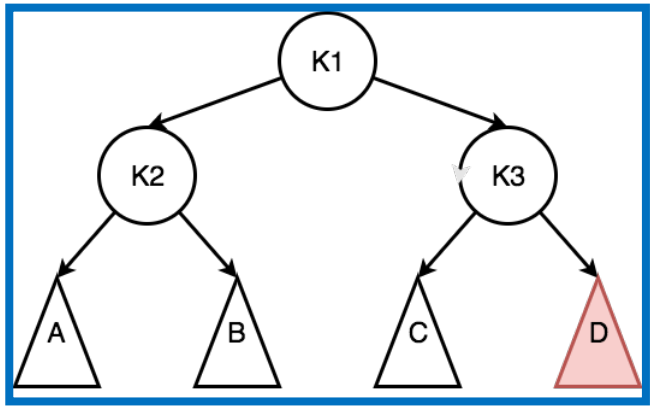
LR 型



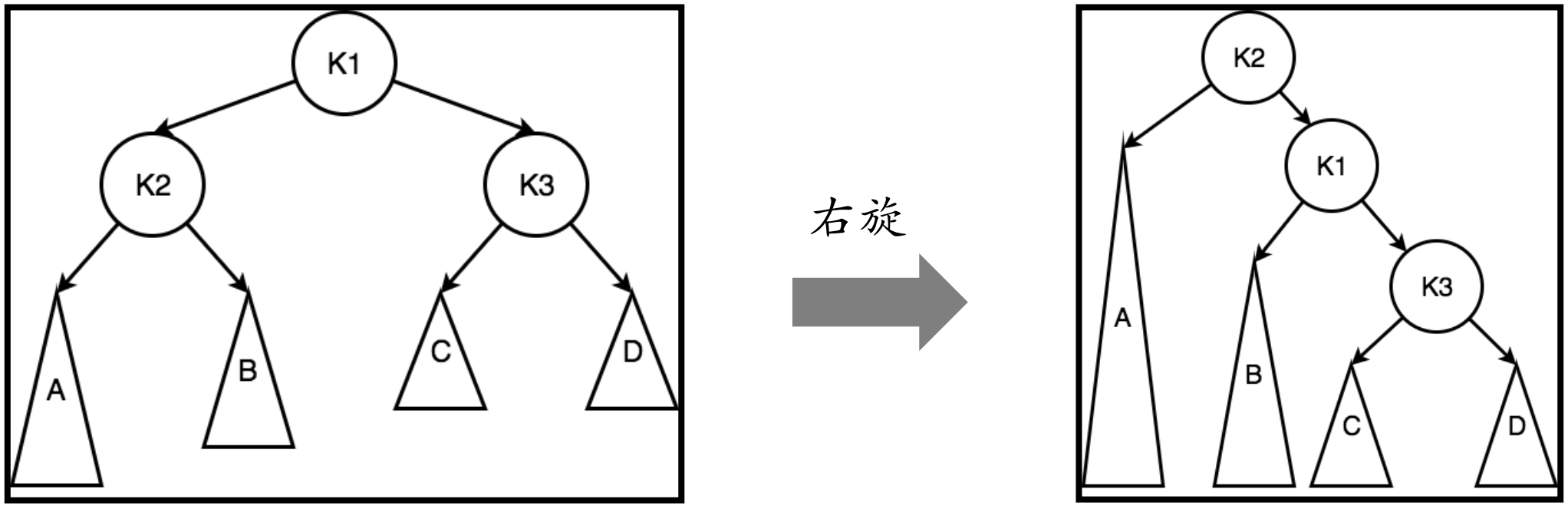
RL 型



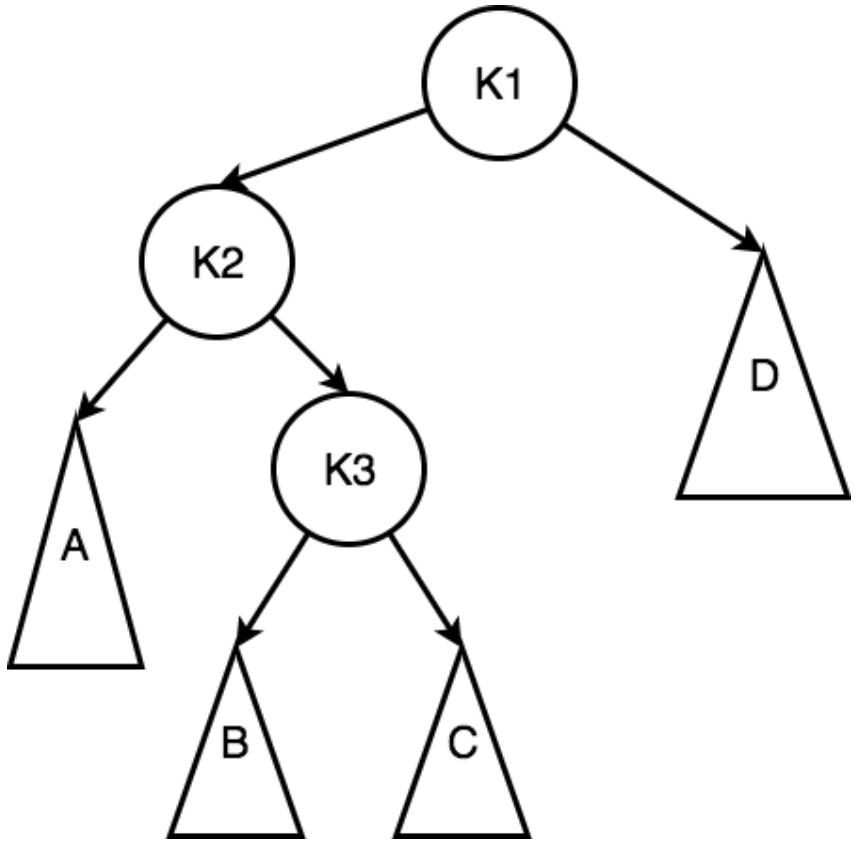
RR 型



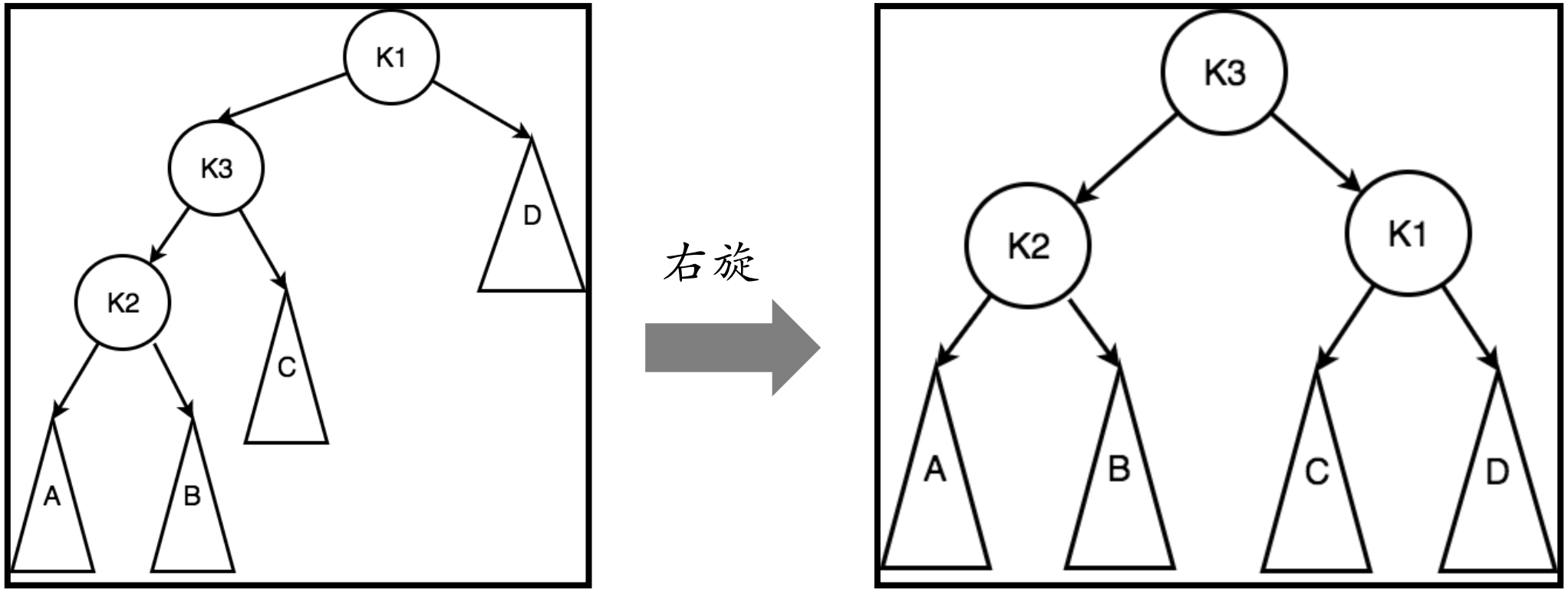
AVL 树-LL 型



AVL 树-LR 型



AVL 树-LR 型-先左旋

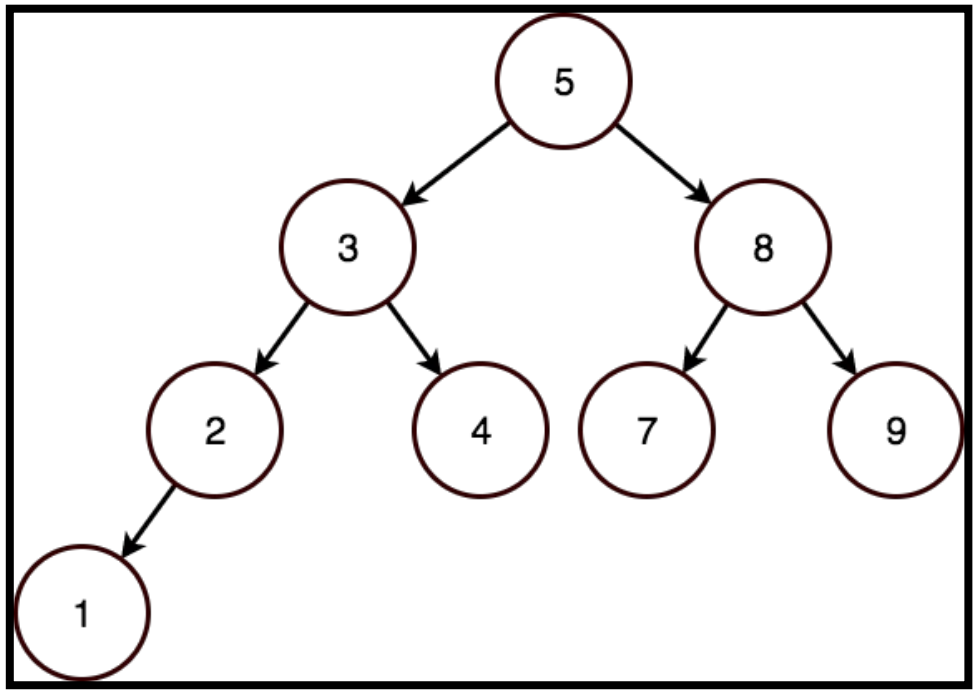


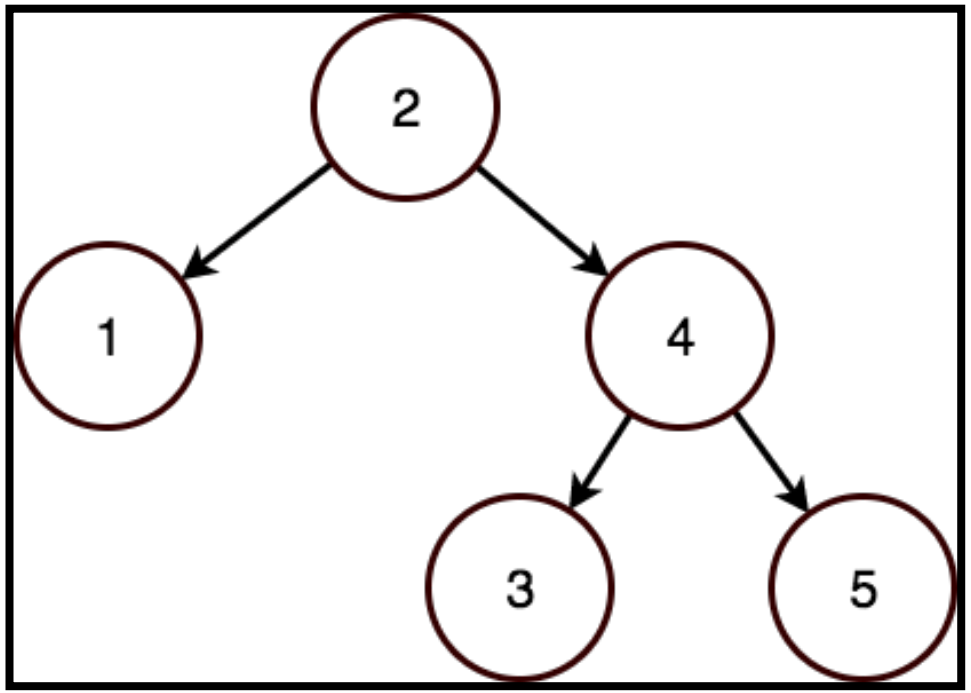
随堂练习-2

请按照如下顺序插入数字，画出对应的 AVL 树

```
1 : [ 5 9 8 3 2 4 1 7 ]
2 : [ 1 2 3 4 5 ]
```

随堂练习-2






1. vim
vim ⌘1
bash ⌘2
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39 }
40
41 Node *insert_maintain(Node *root) {
42 if (!hasRedChild(root)) return root;
43 if (root->lchild->color == RED && root->rchild->color == RED) {
44 if (!hasRedChild(root->lchild) && !hasRedChild(root->rchild)) return root;
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47 return root;
48 }
49 if (root->lchild->color == RED) {
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53 } else {
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56 }
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58 }
59
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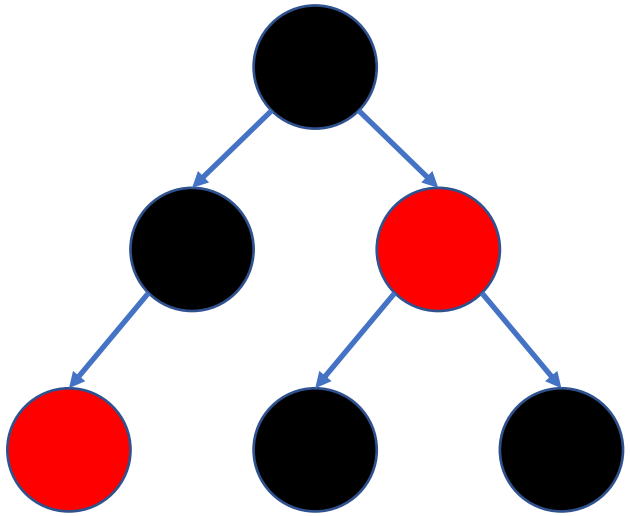
61 Node *__insert(Node *root, int key) {
62 if (root == NIL) return getNewNode(key);
<-6班 资 料 /X.现 场 撸 代 码 /15.RBT.cpp [FORMAT=unix] [TYPE=CPP] [POS=54,30][62%] 21/09/19 - 20:21

AVL树：代码演示

三. 红黑树

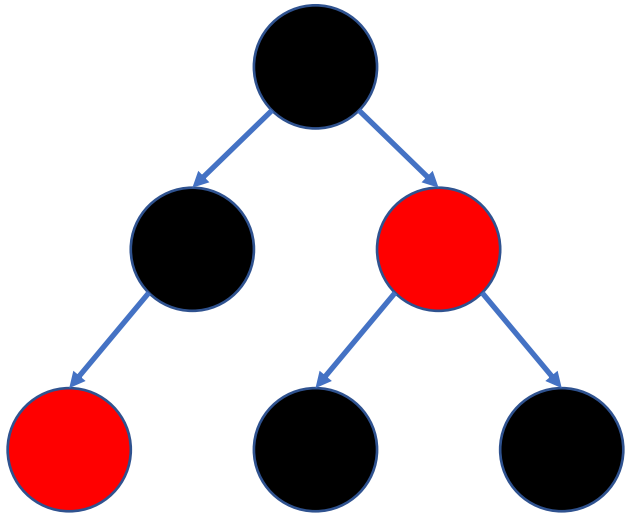
红黑树的平衡条件



1. 每个节点非黑即红
2. 根节点是黑色
3. 叶节点（NIL）是黑色
4. 如果一个节点是红色，则它的两个子节点都是黑色的
5. 从根节点出发到所有叶节点路径上，黑色节点数量相同

红黑树的平衡条件

问题1：红黑树中，最长路径和最短路径长度的关系？



红黑树的平衡条件

问题1：红黑树中，组长路径和最短路径长度的关系？

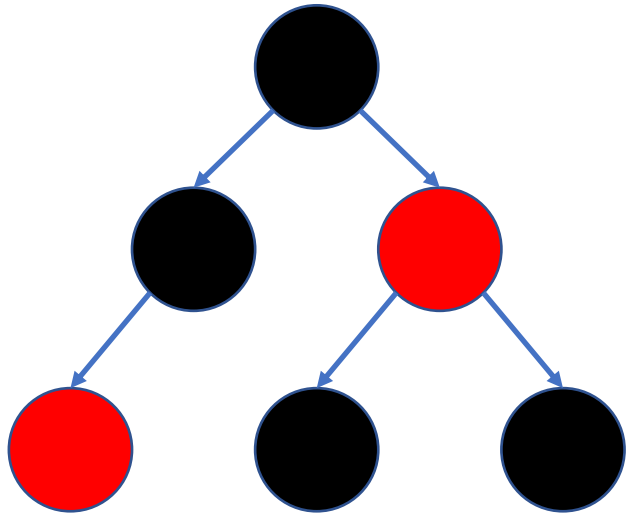
答：

根据平衡条件第4、5两点

最短路径，都是黑色

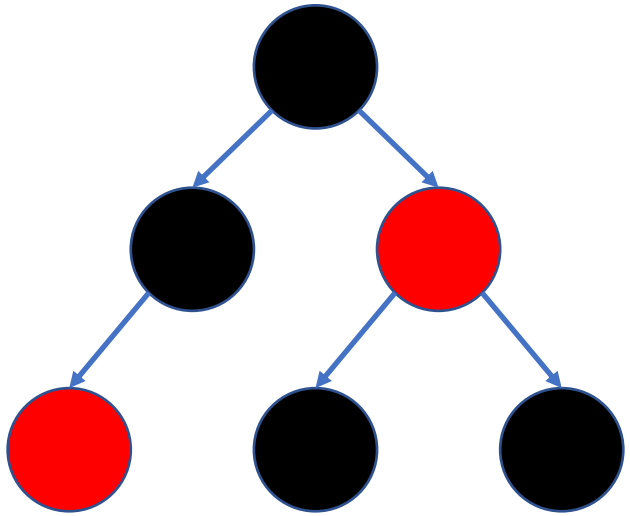
最长路径，红黑相间

最长是最短的两倍



红黑树的平衡条件

问题2：怎么理解条件 3 中的 NIL 节点



红黑树的平衡条件

问题2：怎么理解条件 3 中的 NIL 节点

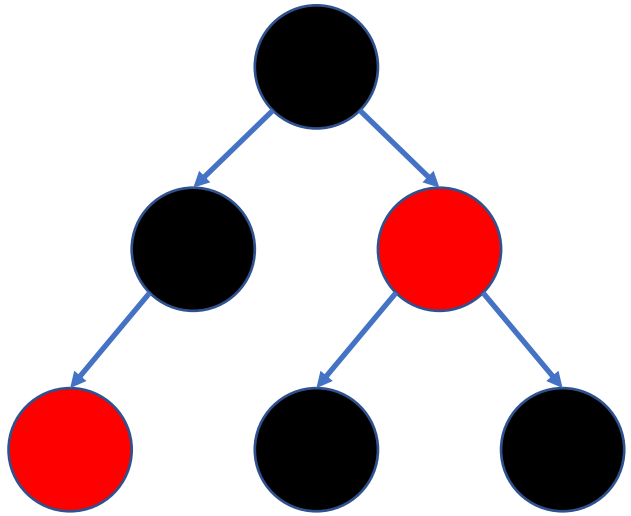
答：

就像文章中的标点符号，

虽然它不属于内容的部分，

平时你也不会注意他，

可要是真没有，就会很麻烦。

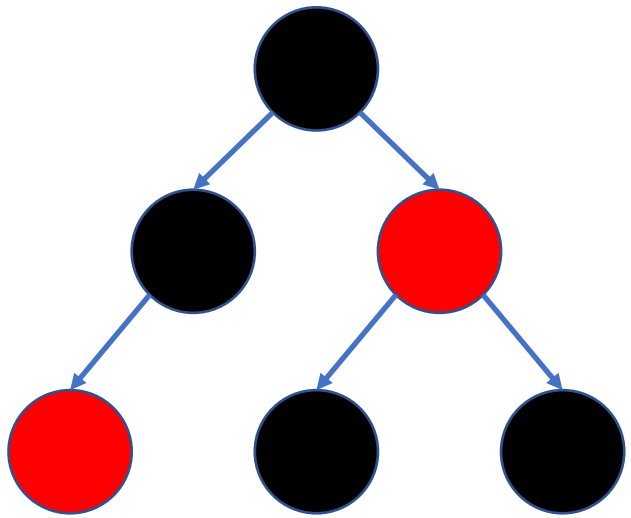


平衡调整终极法门

插入调整站在 祖父节点 看

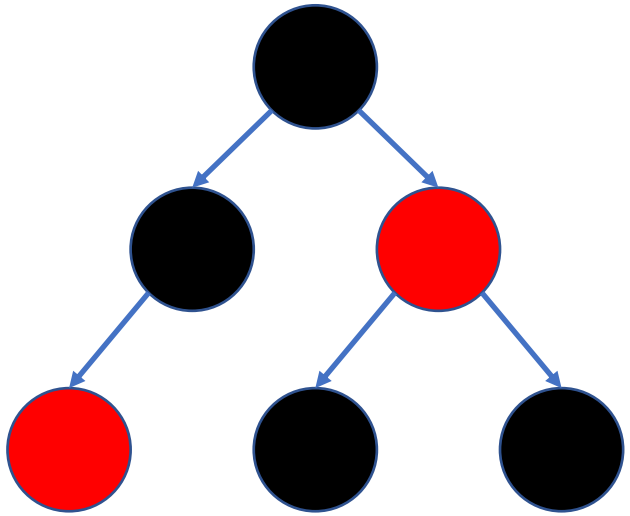
删除调整站在 父节点 看

插入和删除的情况处理一共五种



插入调整的发生场景

问题3：新插入的节点是什么颜色的？红色，黑色？



插入调整的发生场景

问题3：新插入的节点是什么颜色的？红色，黑色？

答：

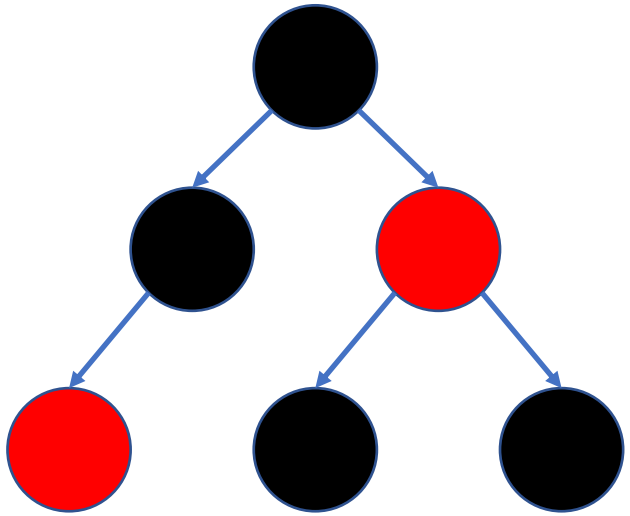
红色，因为

插入黑色一定引发失衡，

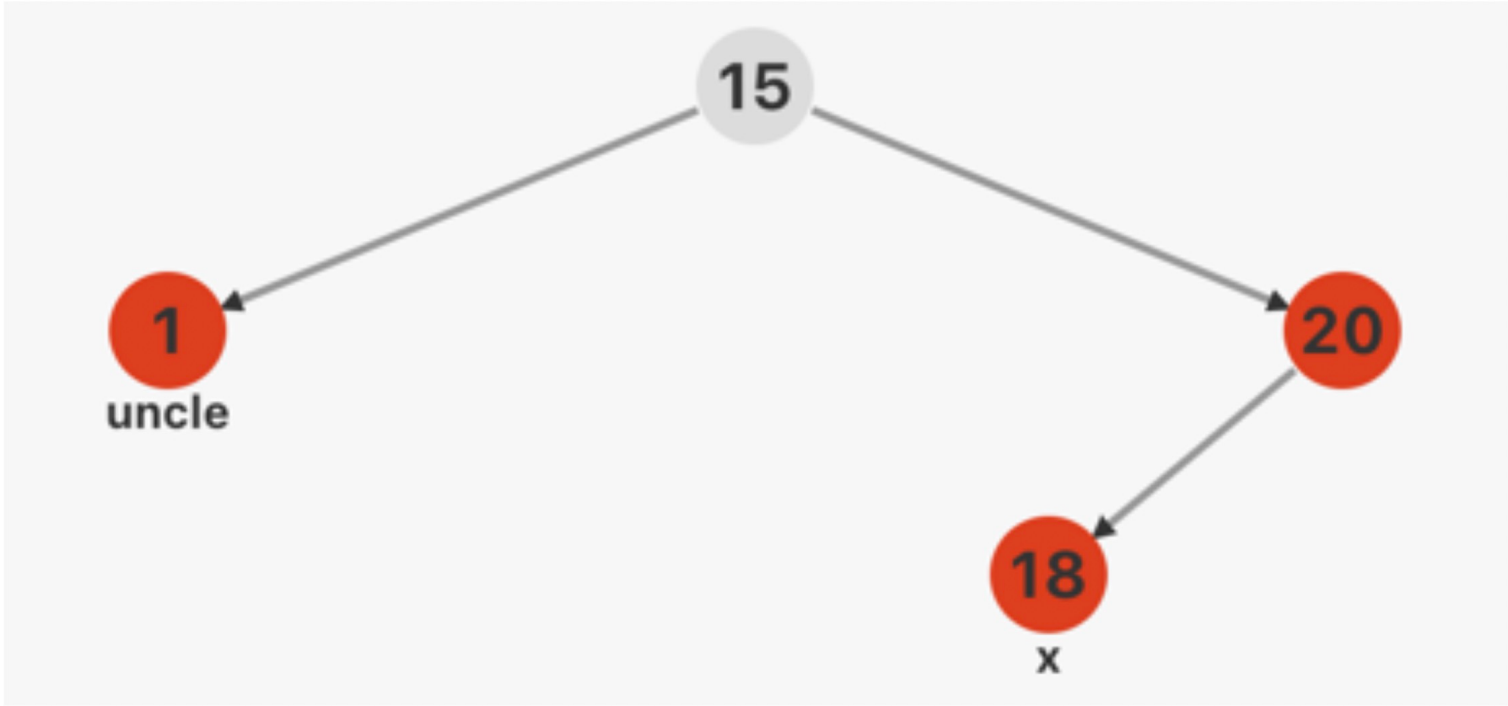
插入红色不一定引发失衡，

一个必死，一个有概率活，

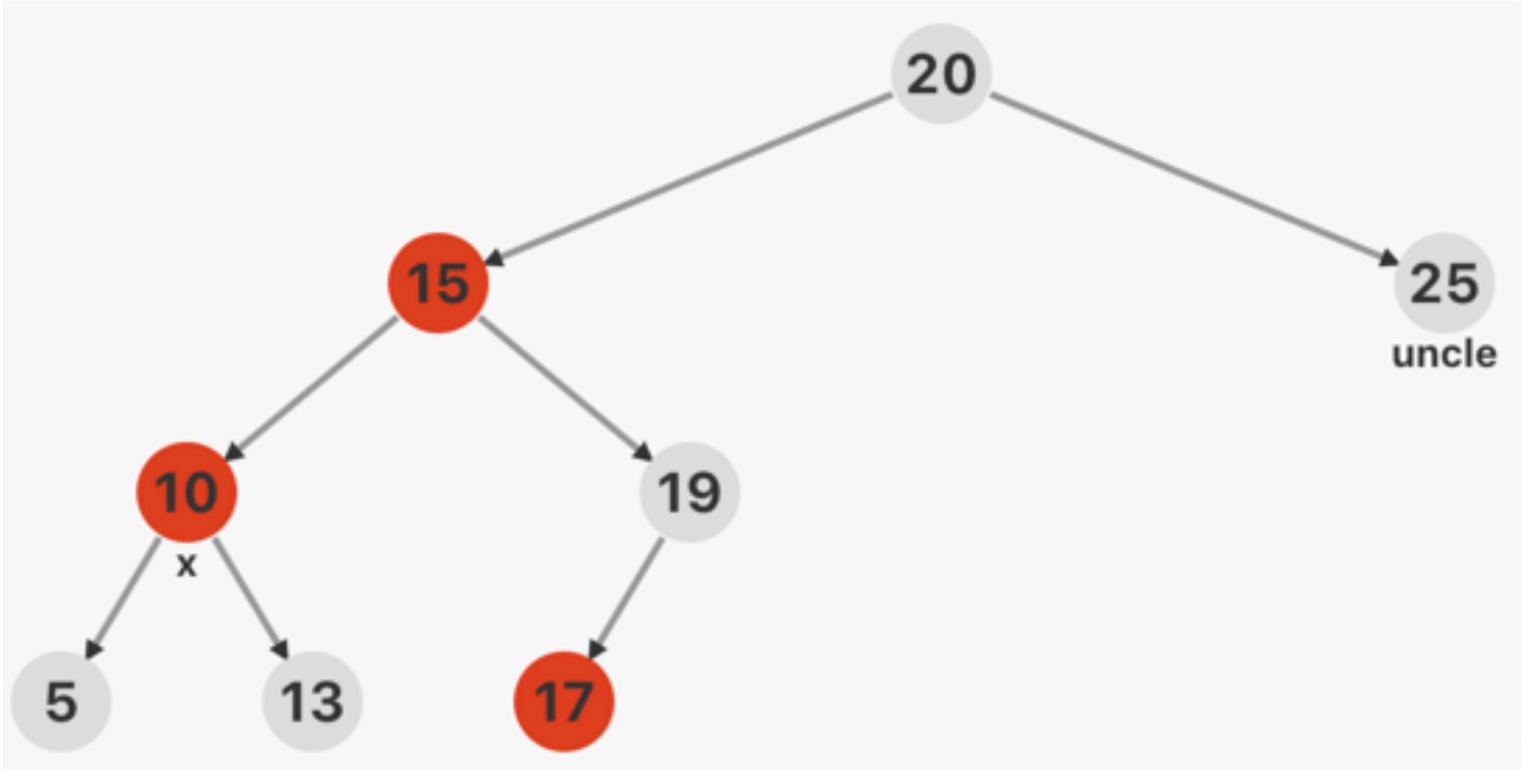
正常人，都会选后者。



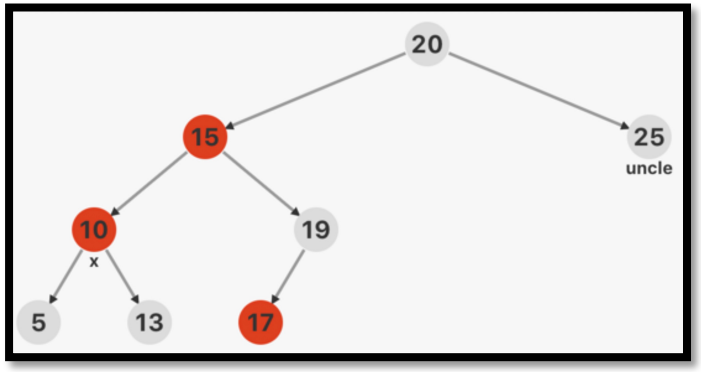
插入调整情况一

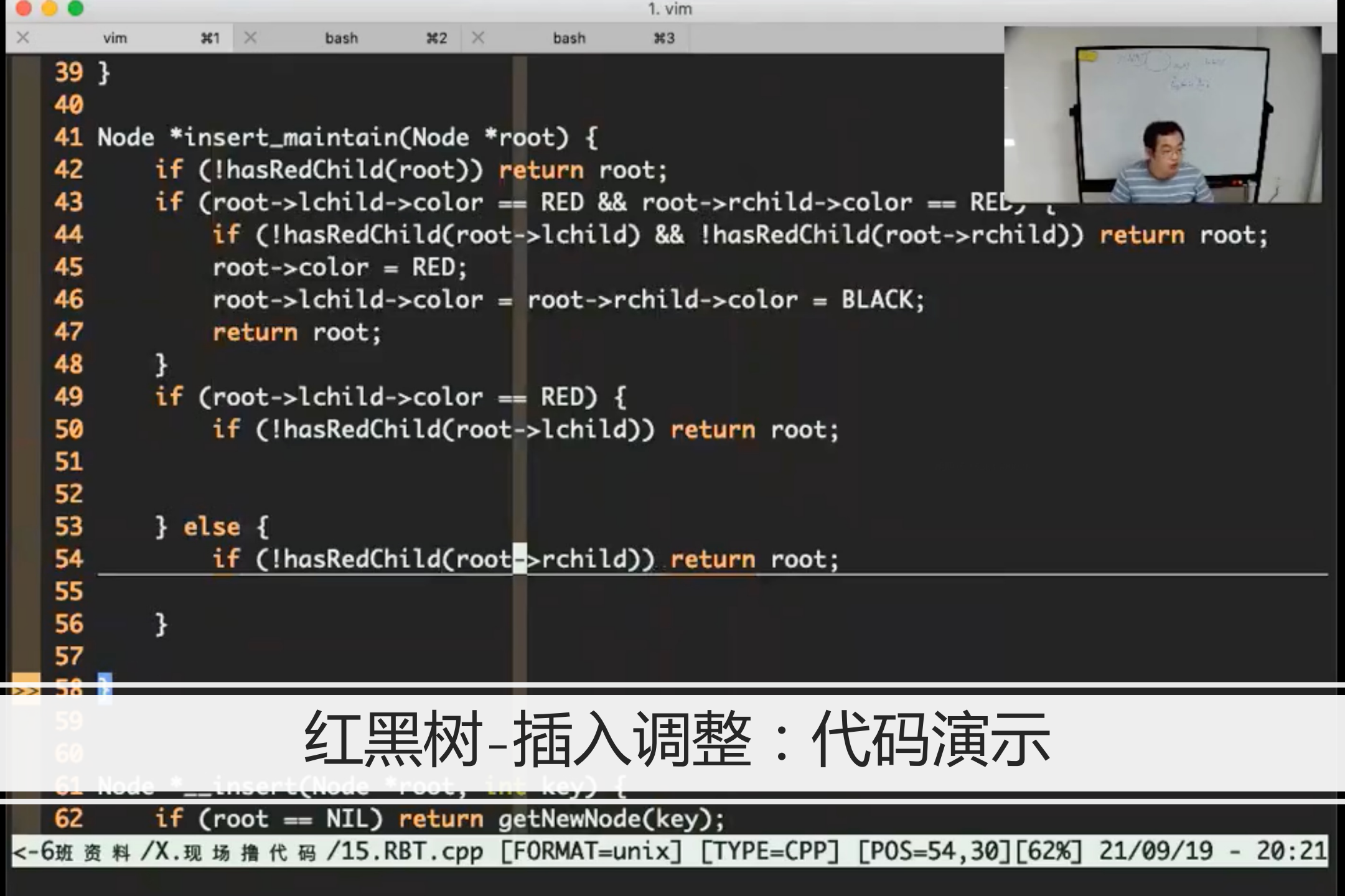


插入调整
情况二



插入调整情况二

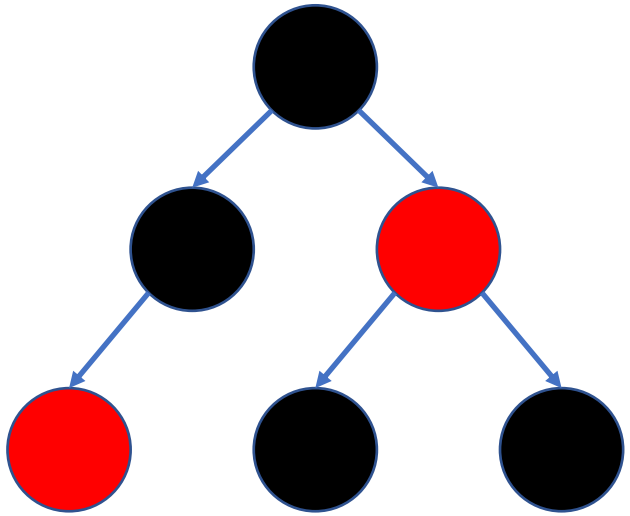




红黑树-插入调整：代码演示

删除调整的发生场景

问题4：删除什么样的节点，会引发红黑树的失衡?



删除调整的发生场景

问题4：删除什么样的节点，会引发红黑树的失衡?

答：

删除度为 0 的黑色节点的时候，

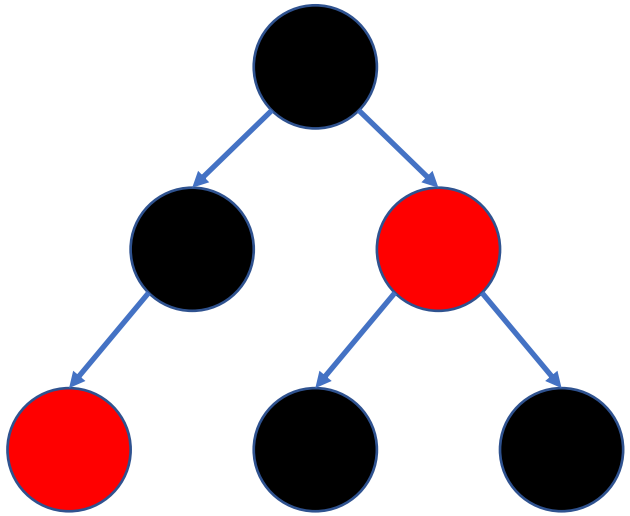
会引发红黑树的失衡。

无处安放的 1 个黑，导致

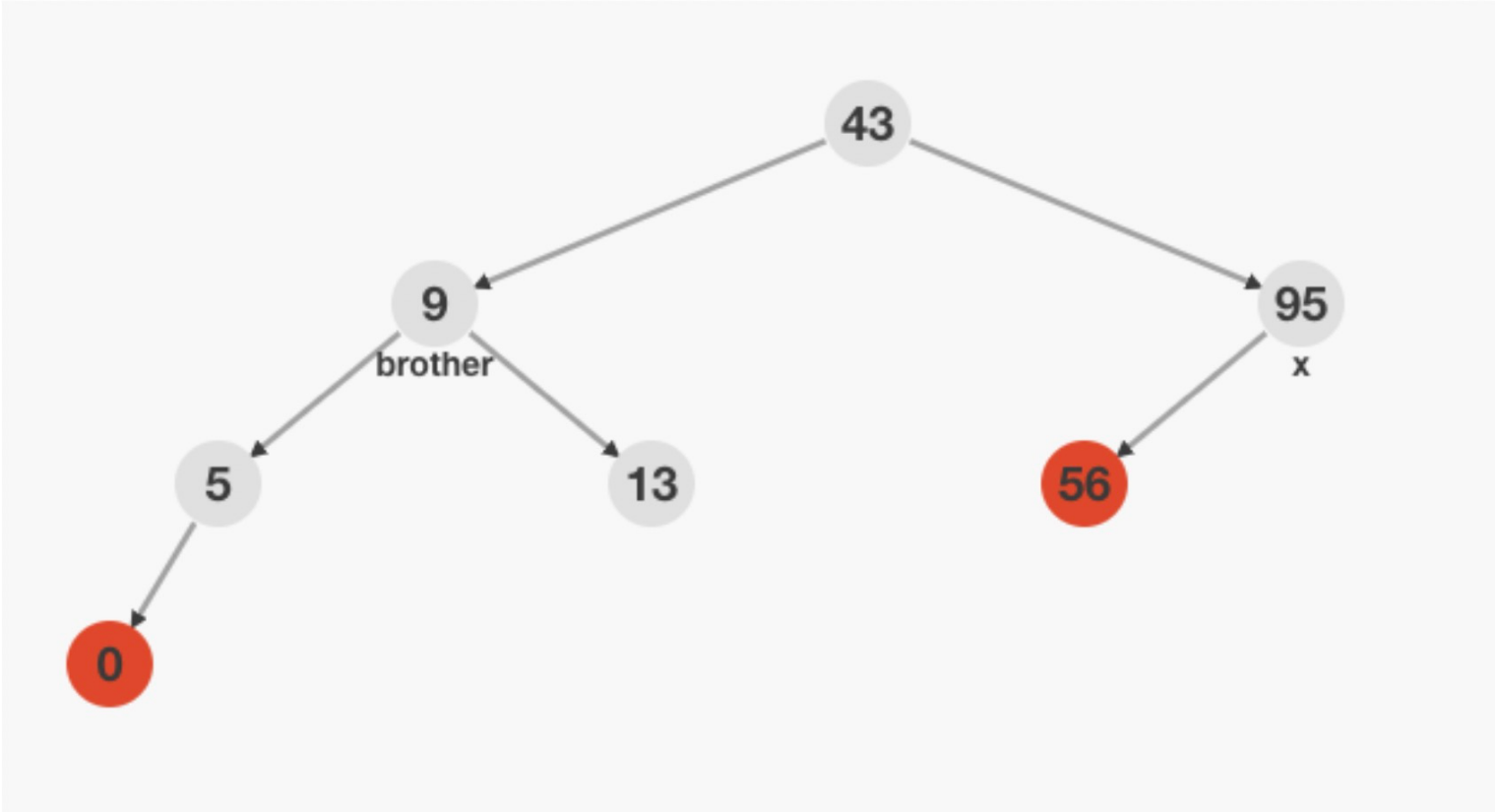
NIL 成了背锅侠。

从此以后， NIL 彻底黑化。

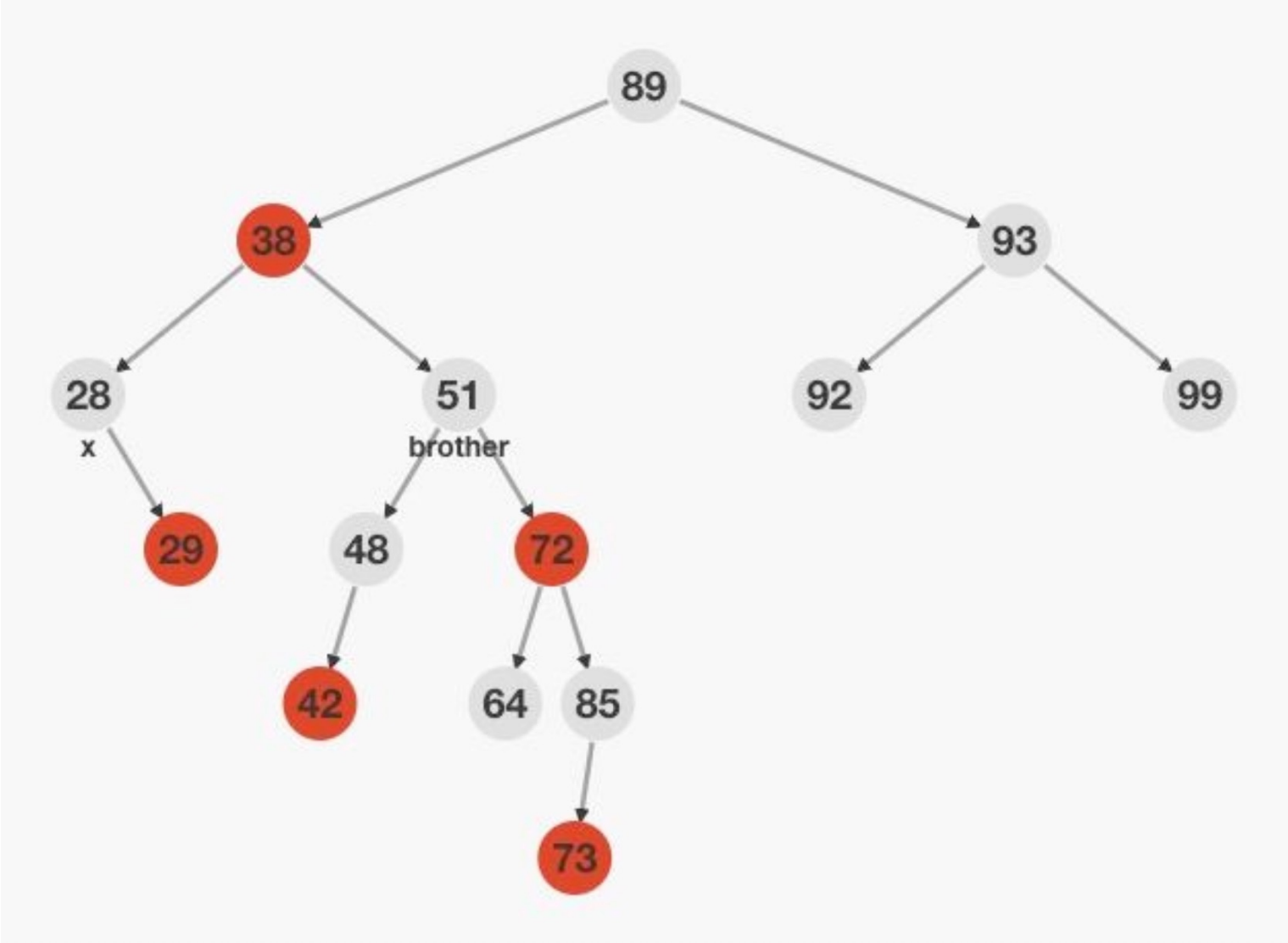
这就是【双重黑】的诞生过程。



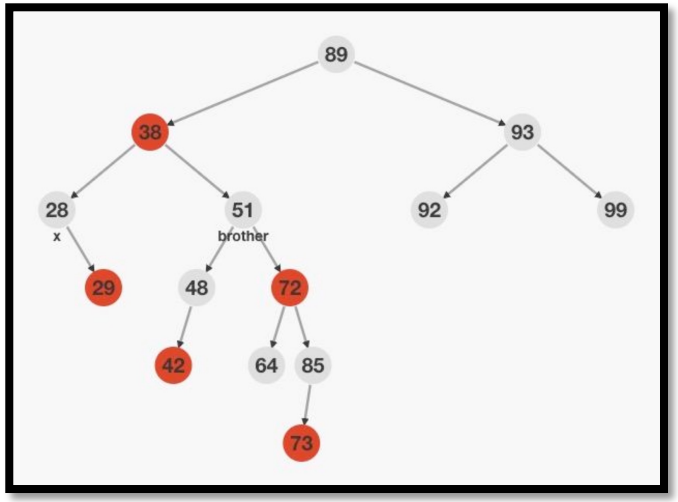
删除调整情况一



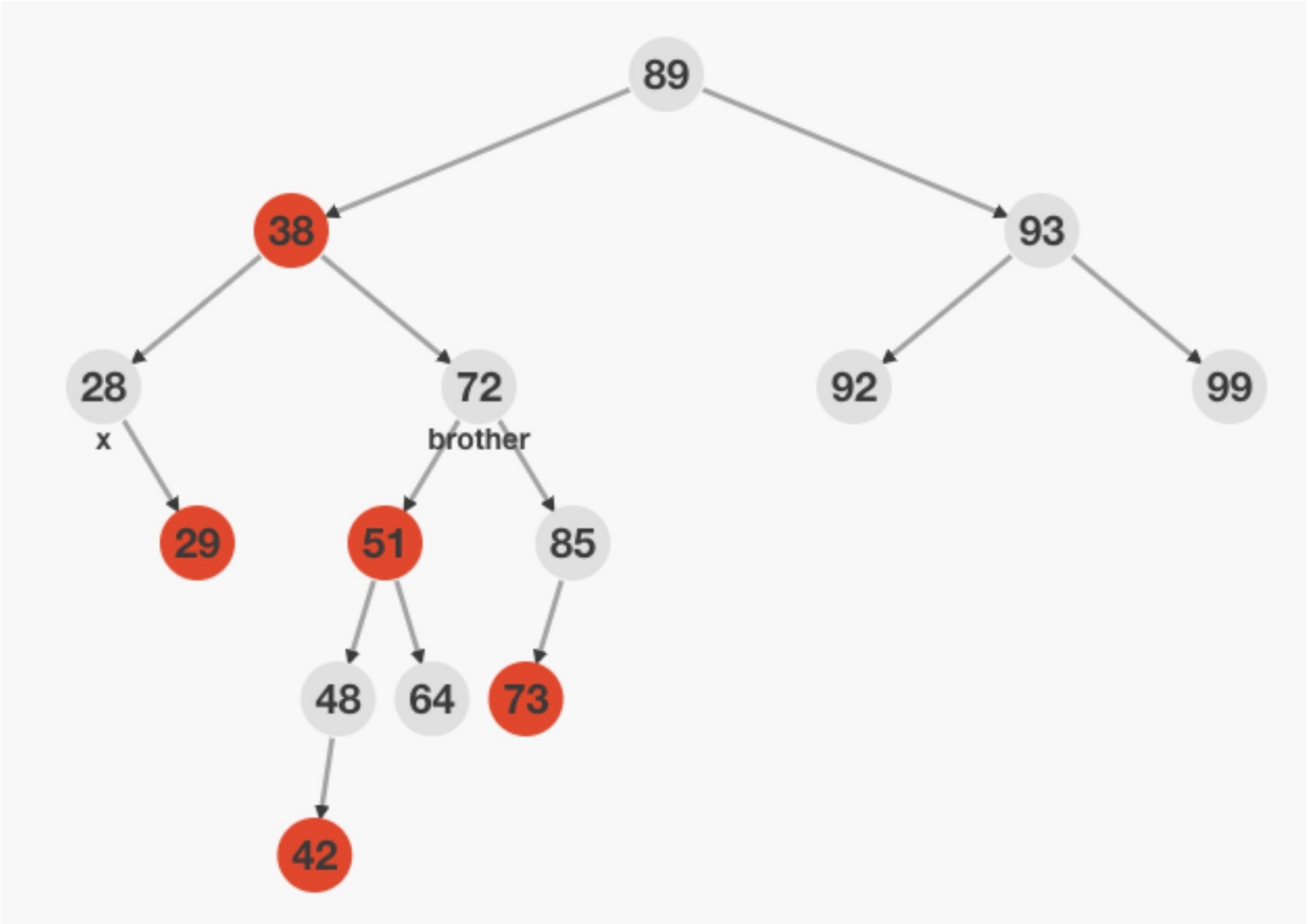
删除调整情况二



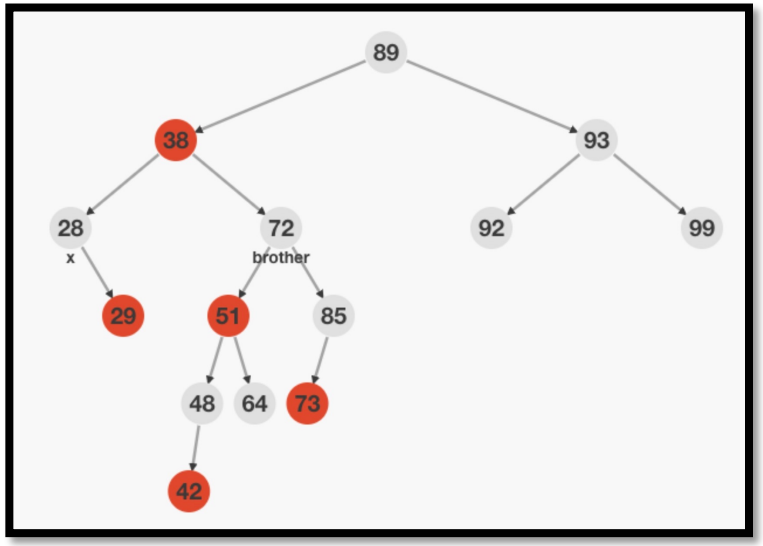
删除调整情况二



删除调整
情况三



删除调整
情况三



红黑树-删除调整：代码演示

四. B-树

B 树 – 结构定义

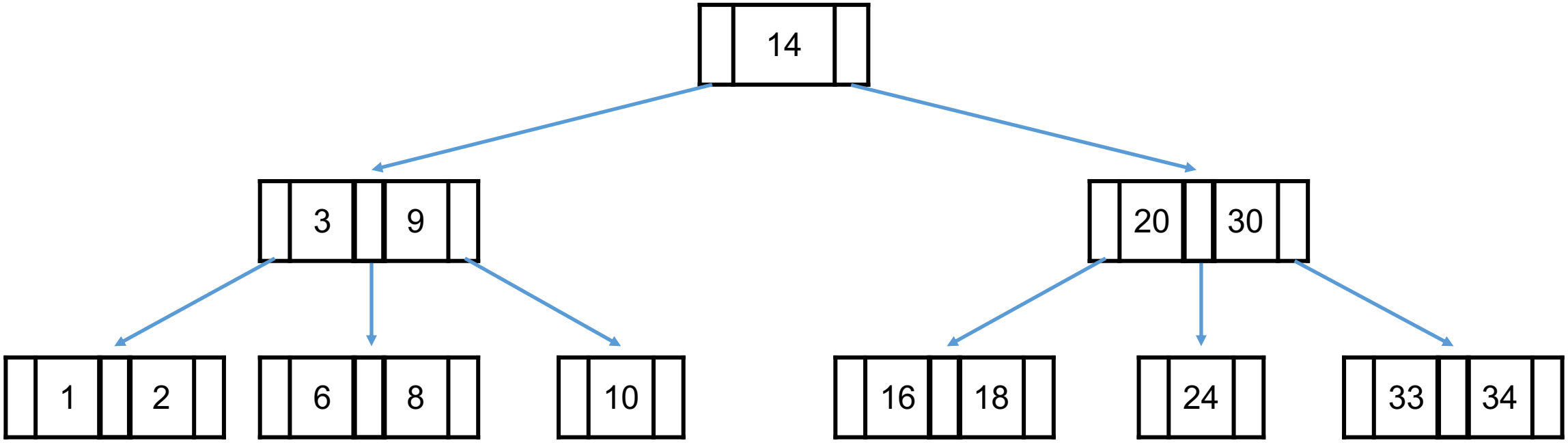
一棵 m 阶 B 树，需要满足下列特性：

1. 树中每个节点，最多含有 m 棵子树
2. 若根节点不是叶子节点，则至少有 2 棵子树
3. 除根结点之外的所有非终端结点至少有[m/2]棵子树
4. 如果一个结点有n-1个关键字，则该结点有n个分支，且这n-1个关键字按照递增顺序排列
5. 每个结点的结构为：（ n，A_0，K_1，A_1，K_2，A_2，… ，K_n，A_n ）
6. 非根结点中关键字的个数 n，满足：[m/2]-1 ≤ n ≤ m-1
7. 所有叶子节点处在同一层

B 树 – 结构定义

m 阶 B 树的性质解读：

1. B 树中只有根节点没办法满足拥有至少 [m/2] 棵子树的条件，其他节点均能满足
2. B 树是一种高度平衡的树形结构，比 AVL 树结构上更优美



1. 元素插入

2. 插入调整

3. 元素删除

4. 删除调整

1. 元素插入

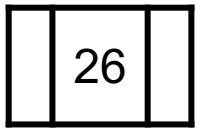
2. 插入调整

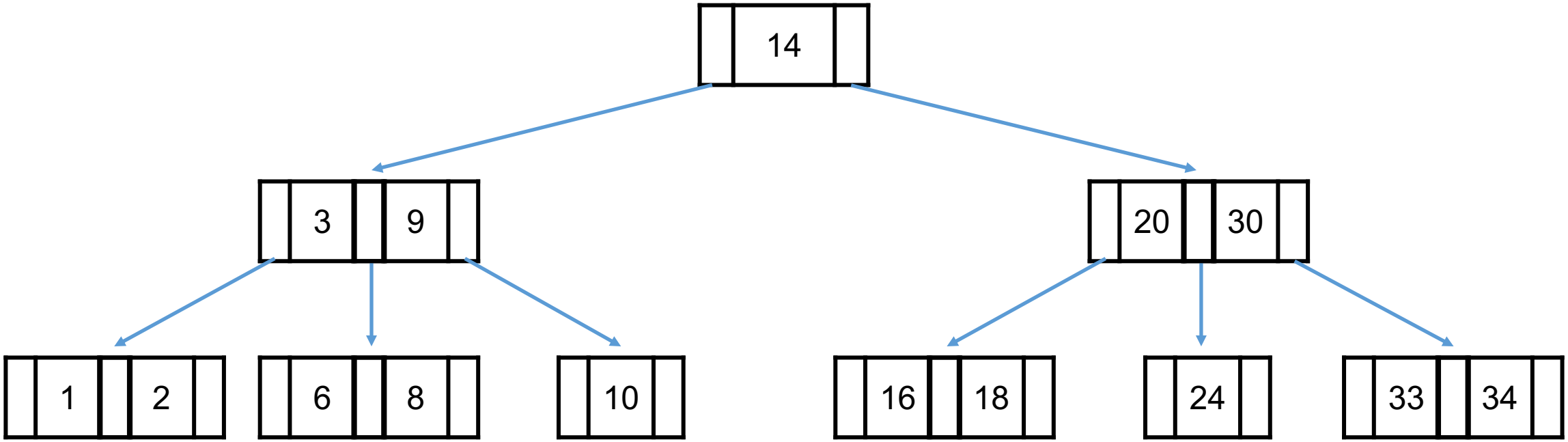
3. 元素删除

4. 删除调整

B 树 – 元素插入

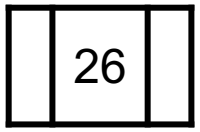
插入：

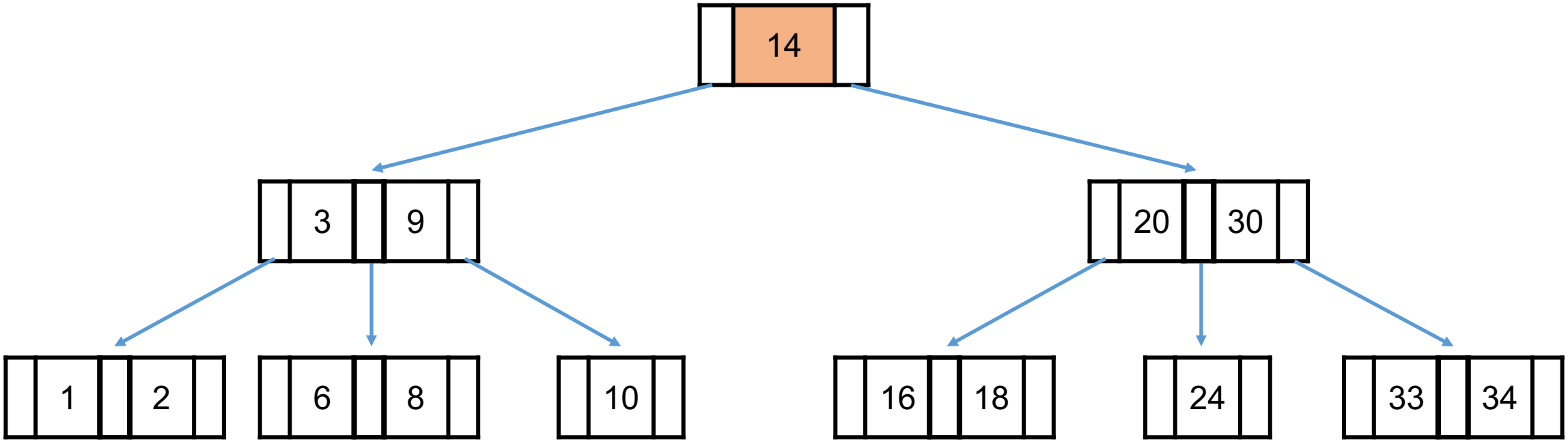




B 树 – 元素插入

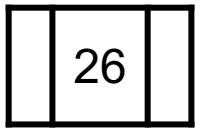
插入：

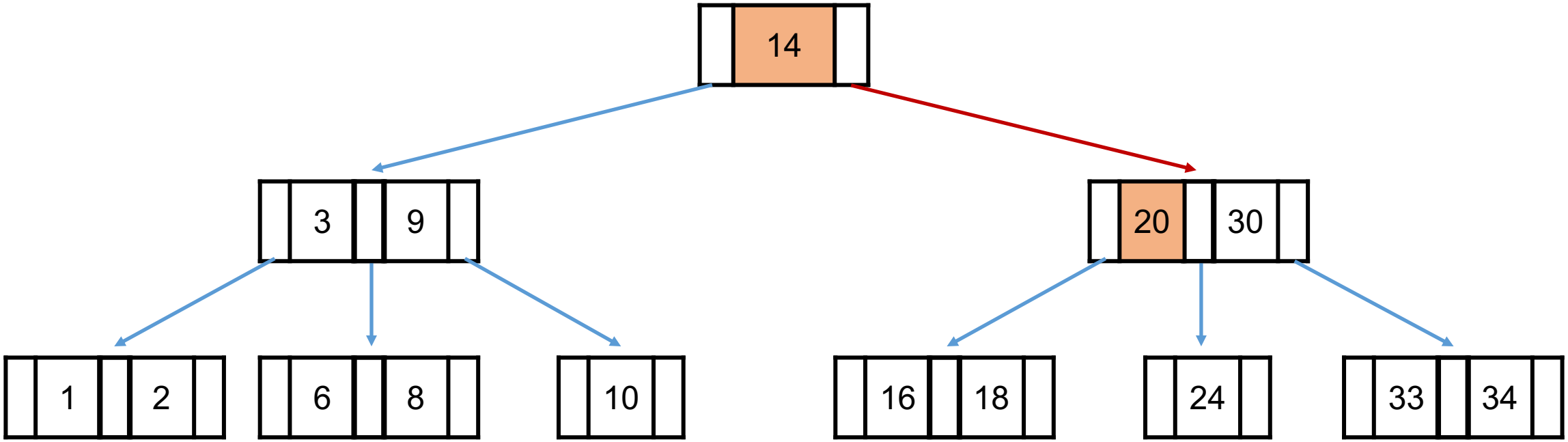




B 树 – 元素插入

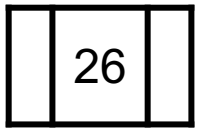
插入：

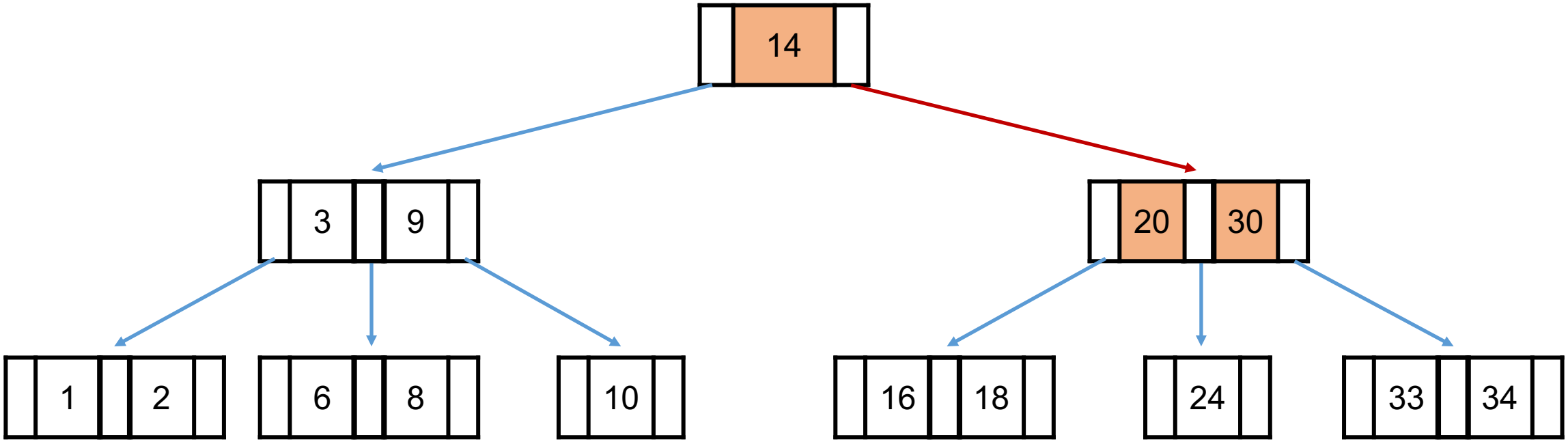




B 树 – 元素插入

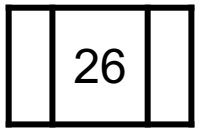
插入：

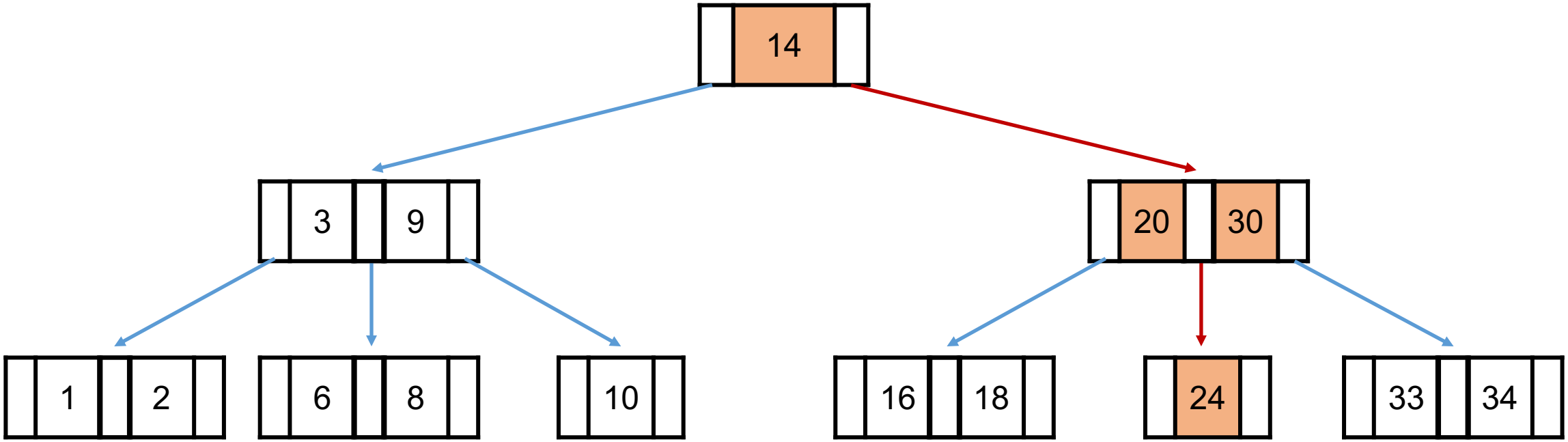




B 树 – 元素插入

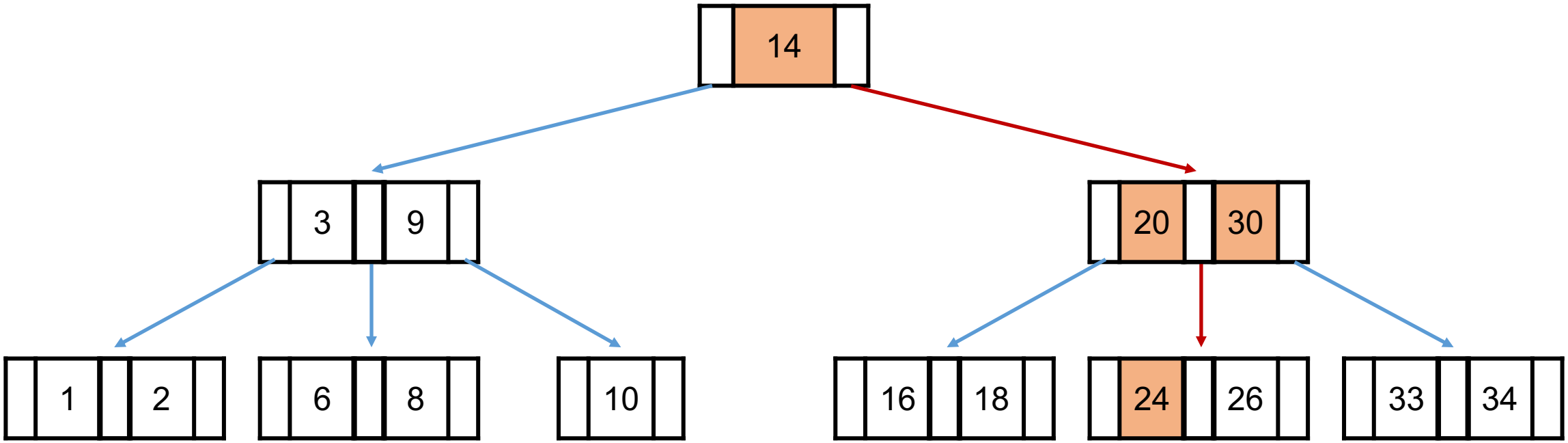
插入：



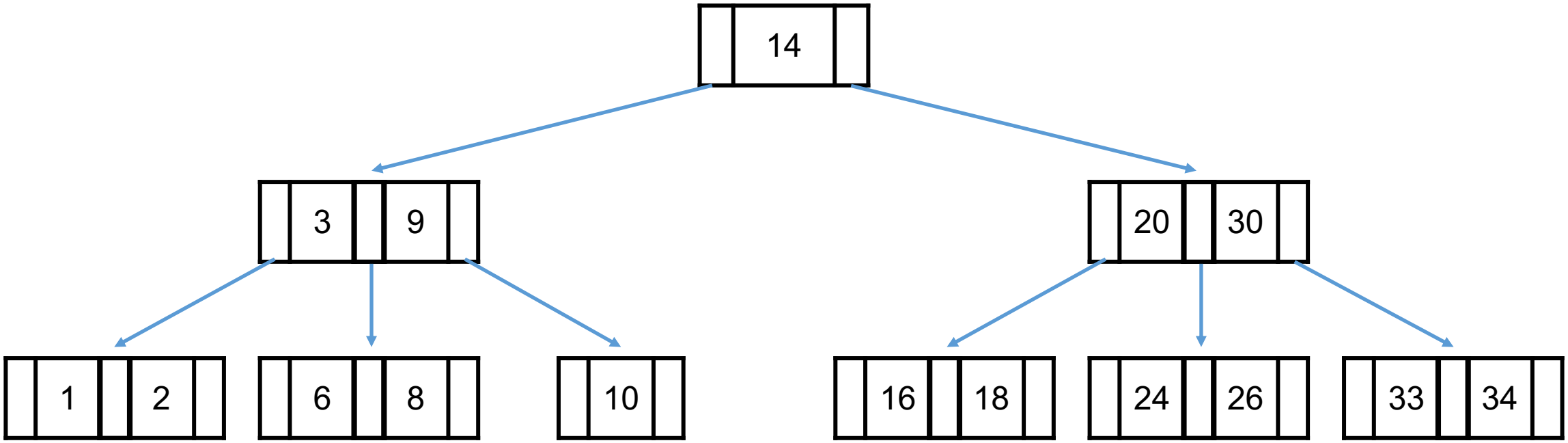


B 树 – 元素插入

插入：



B 树 – 元素插入



~~1. 元素插入~~

2. 插入调整

3. 元素删除

4. 删除调整

~~1. 元素插入~~

2. 插入调整

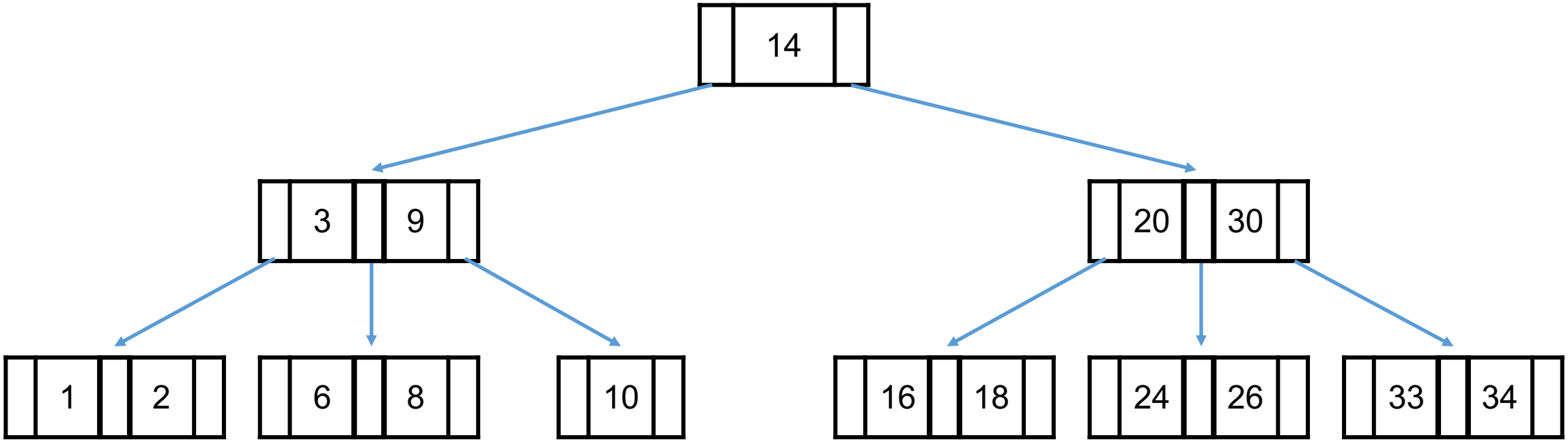
3. 元素删除

4. 删除调整

B 树 – 插入调整（上溢）

m 阶 B 树的插入调整:

1. 插入调整站在父节点处理，发生在节点关键字数量达到 m 时
2. 插入调整的核心操作是: 节点分裂



B 树 – 插入调整（上溢）

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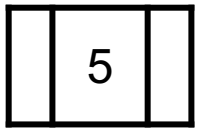
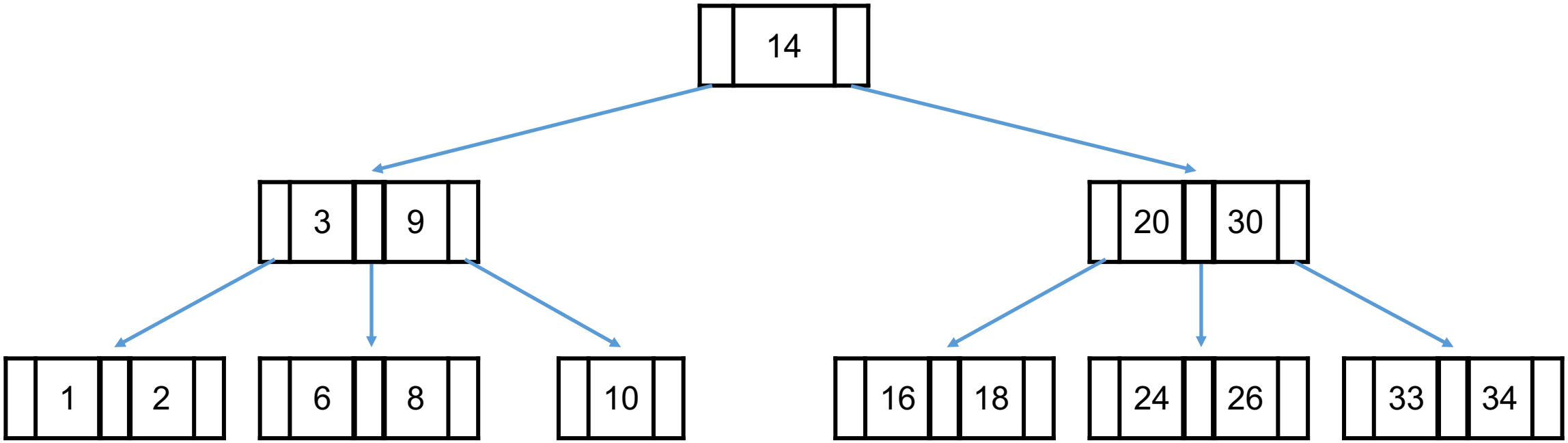
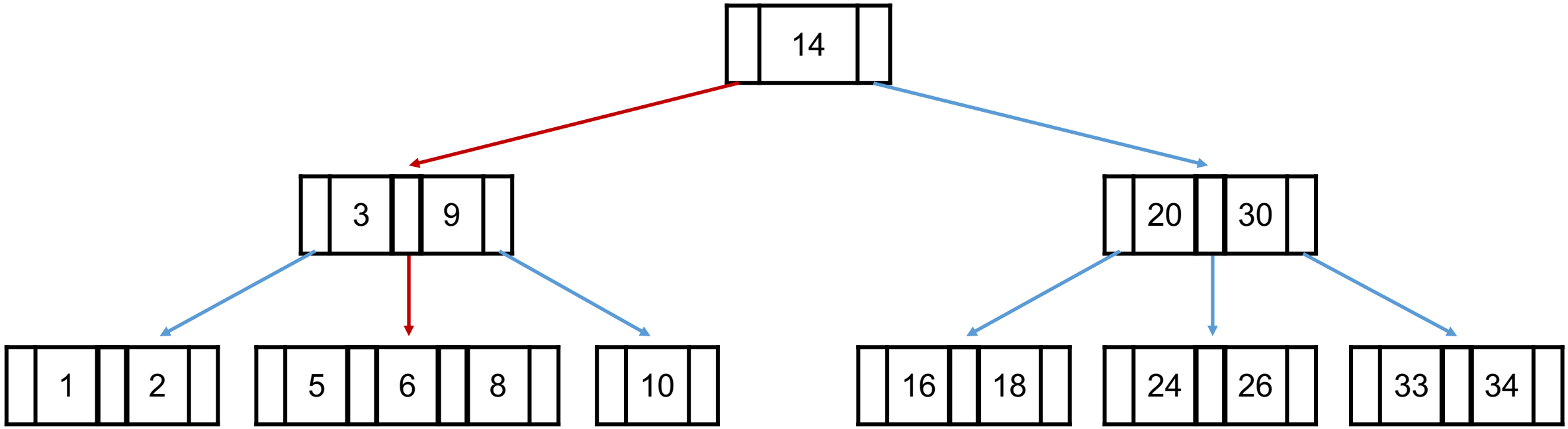




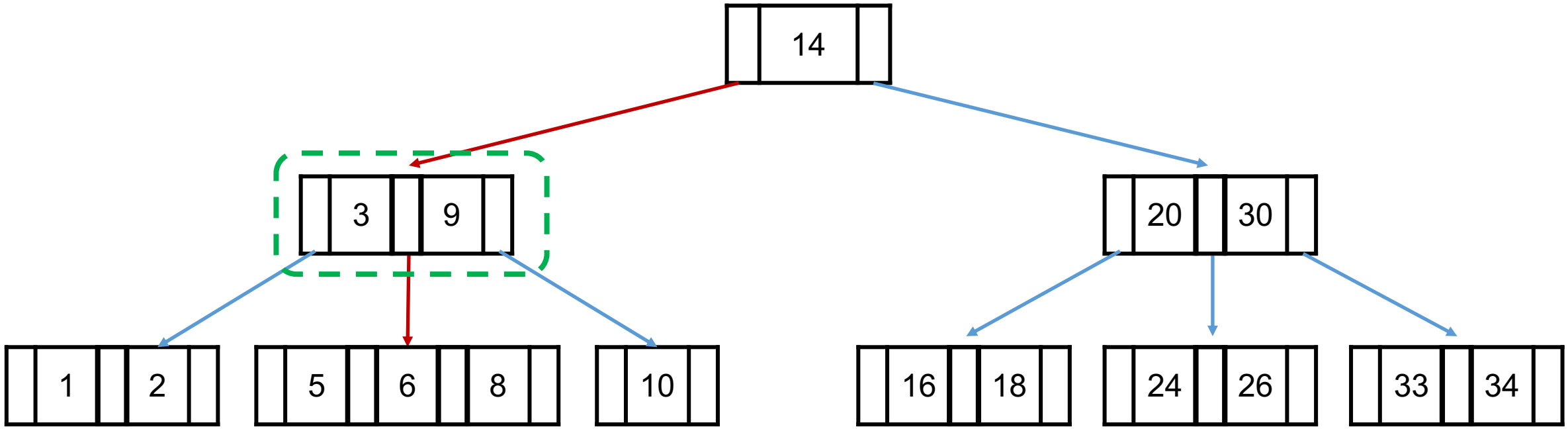
B 树 – 插入调整（上溢）

插入：



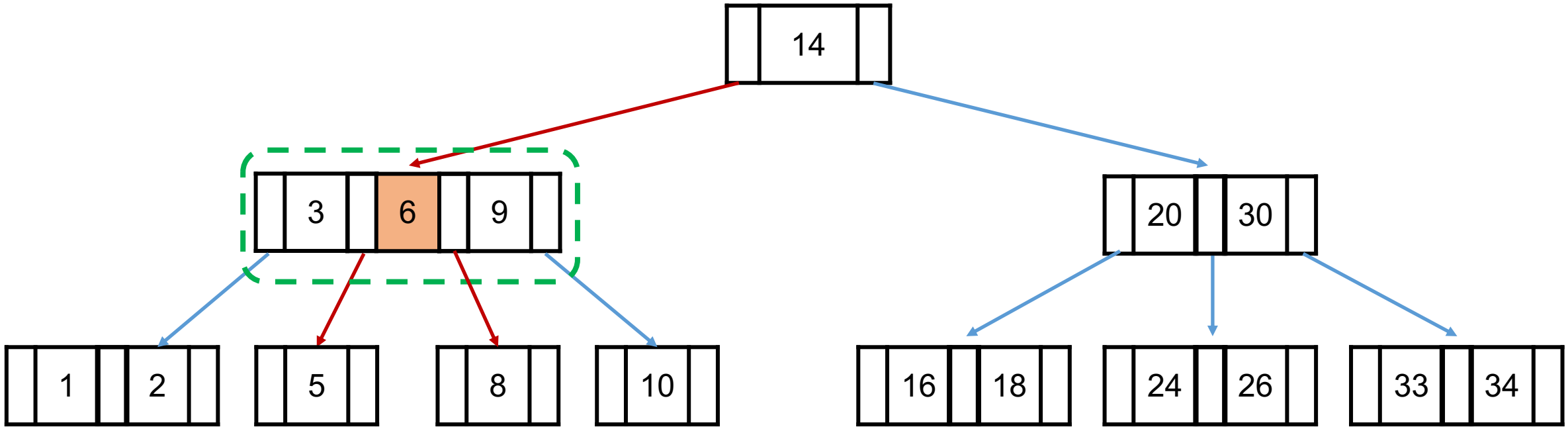
B 树 – 插入调整（上溢）

站在父节点处理



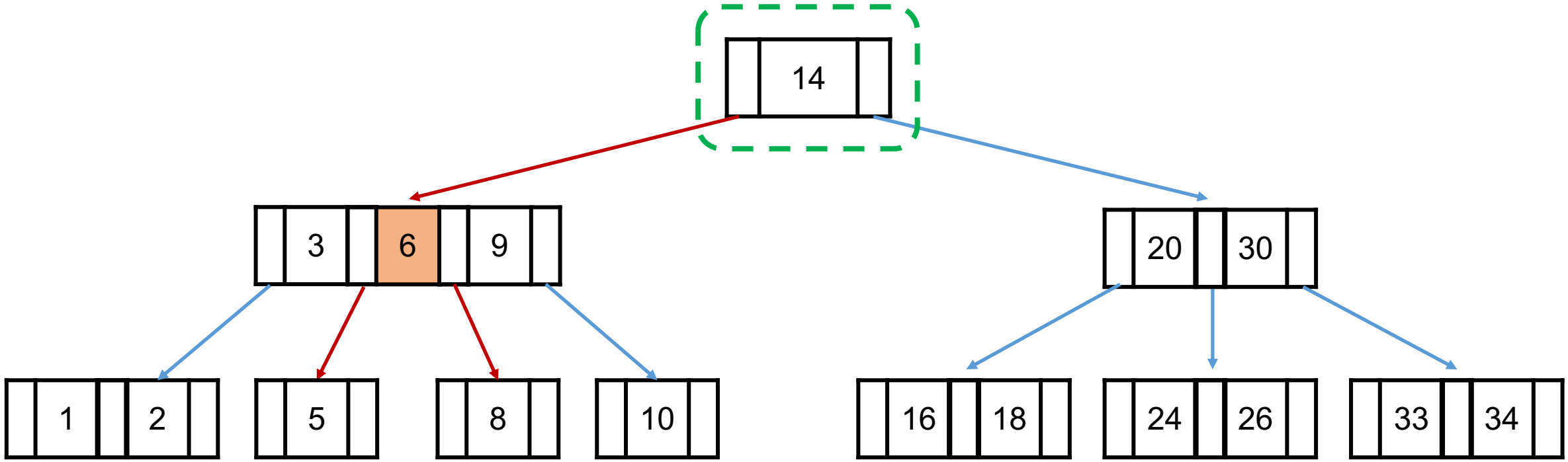
B 树 – 插入调整（上溢）

节点分裂



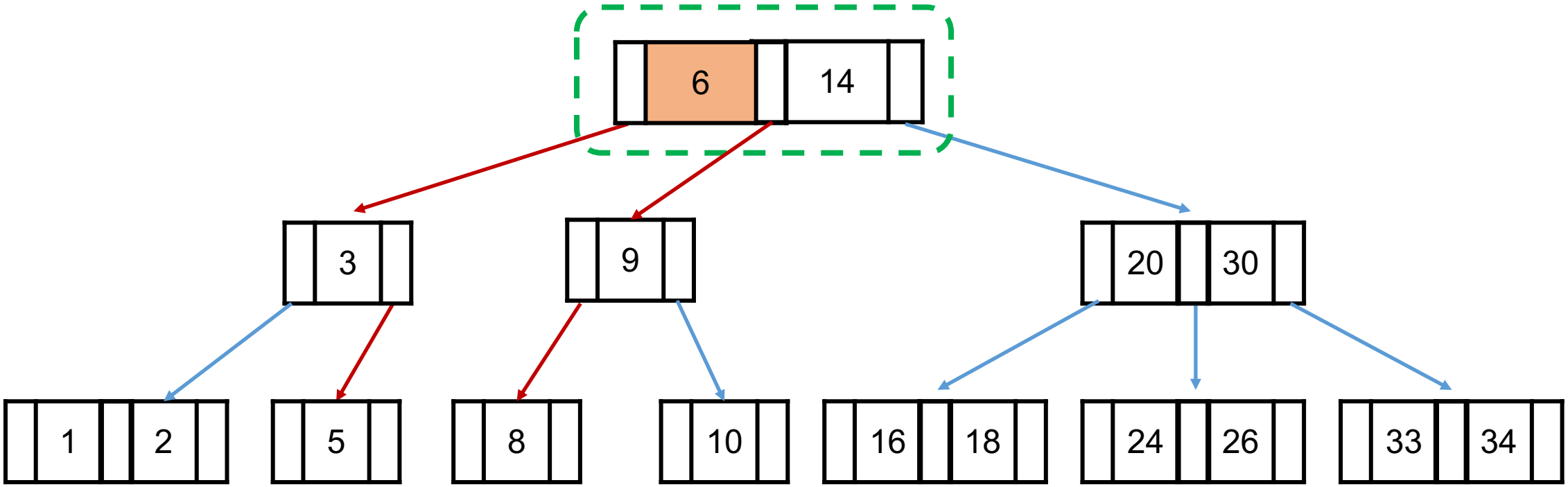
B 树 – 插入调整（上溢）

站在父节点处理

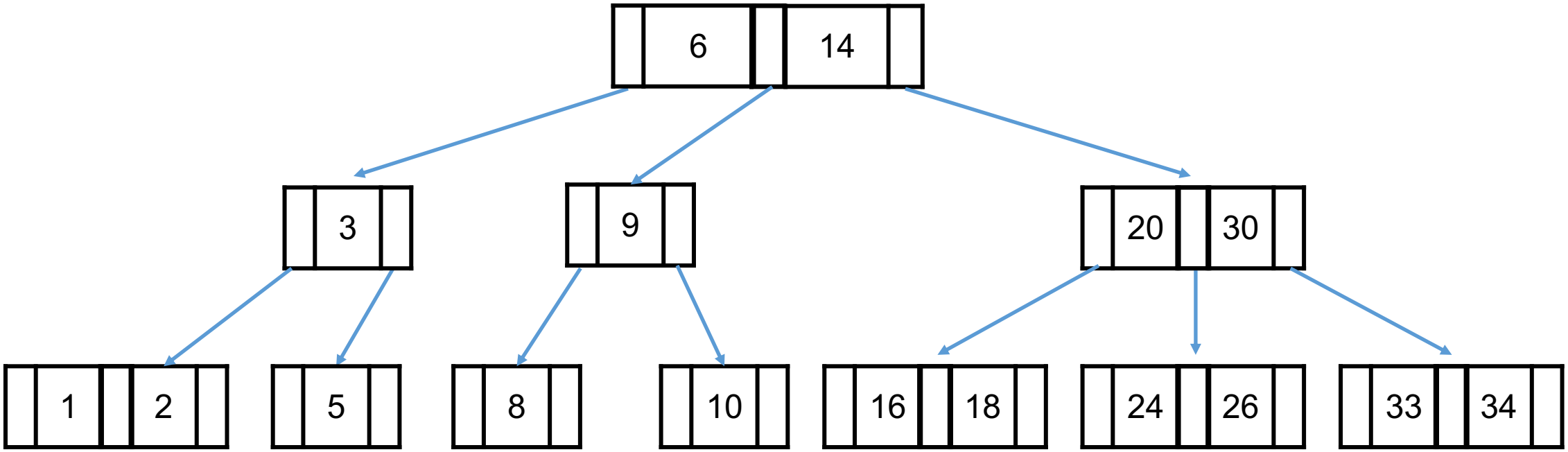


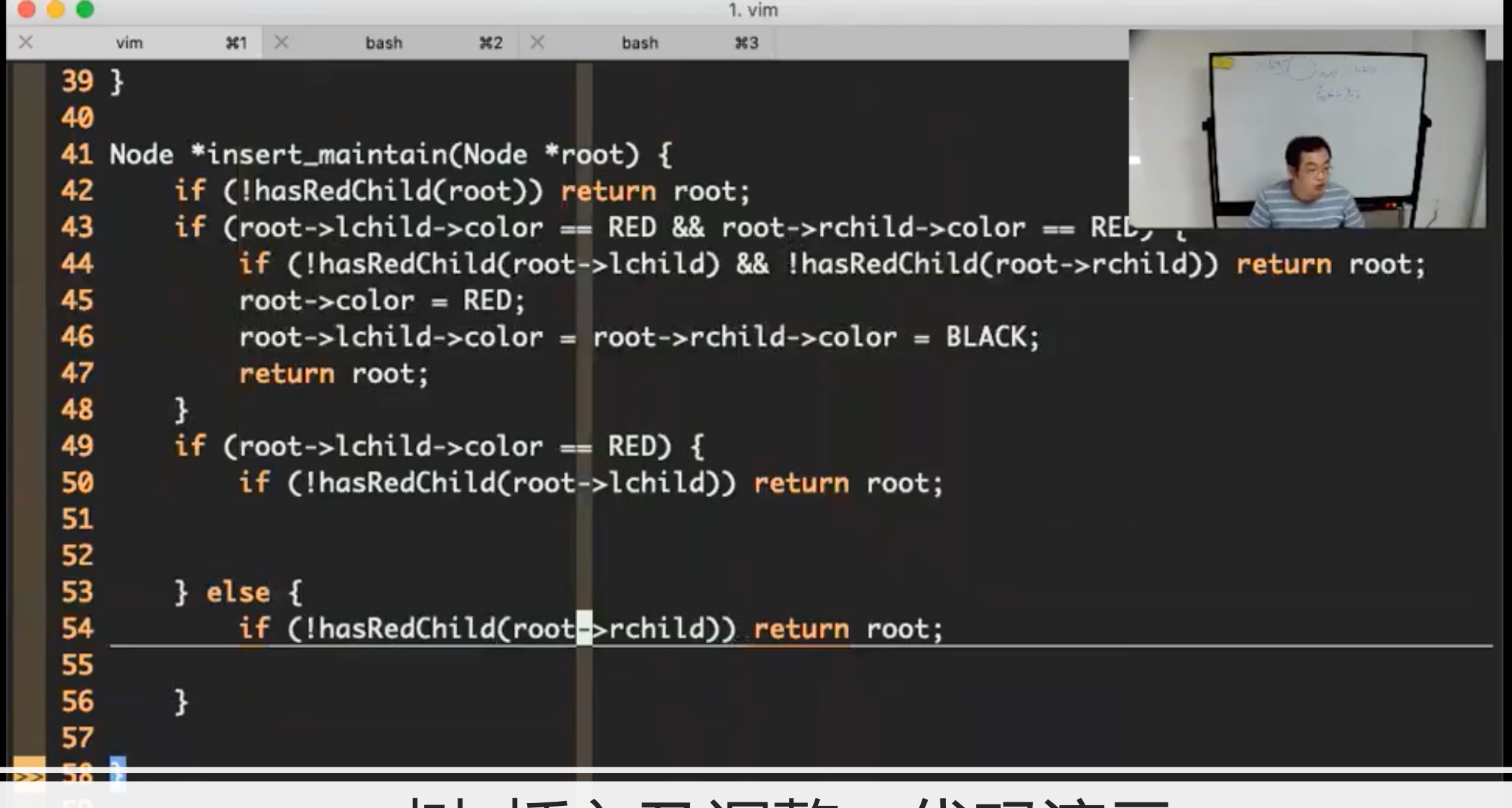
B 树 – 插入调整（上溢）

节点分裂



B 树 – 插入调整（上溢）





B 树-插入及调整：代码演示

~~1. 元素插入~~

~~2. 插入调整~~

3. 元素删除

4. 删除调整

~~1. 元素插入~~

~~2. 插入调整~~

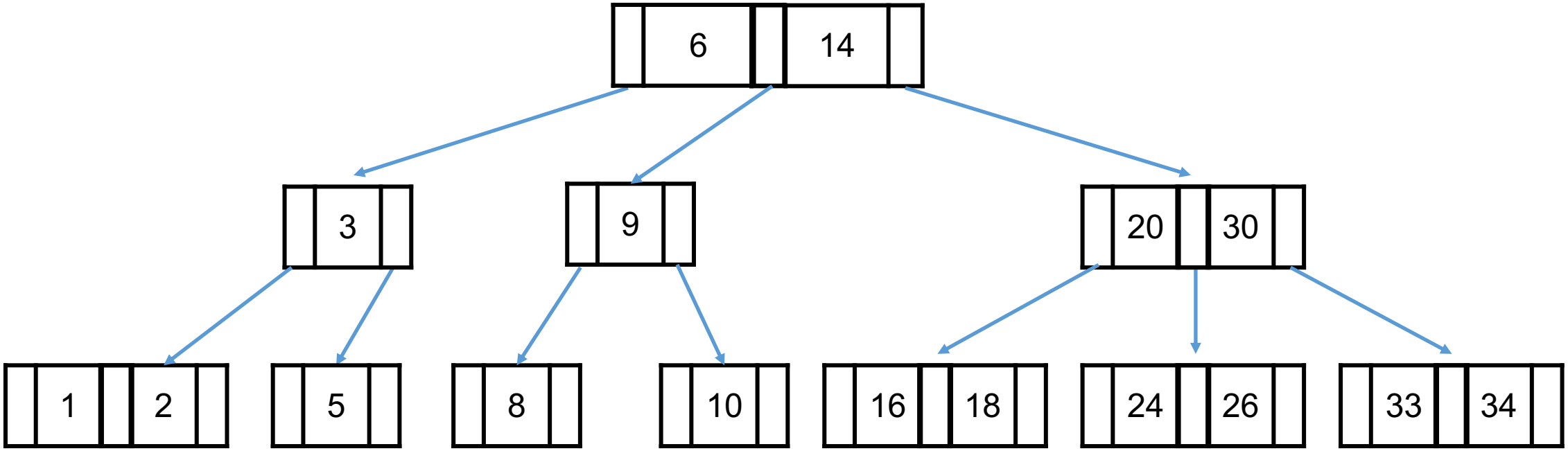
3. 元素删除

4. 删除调整

B 树 – 元素删除(1)

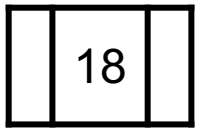
m 阶 B 树的元素删除:

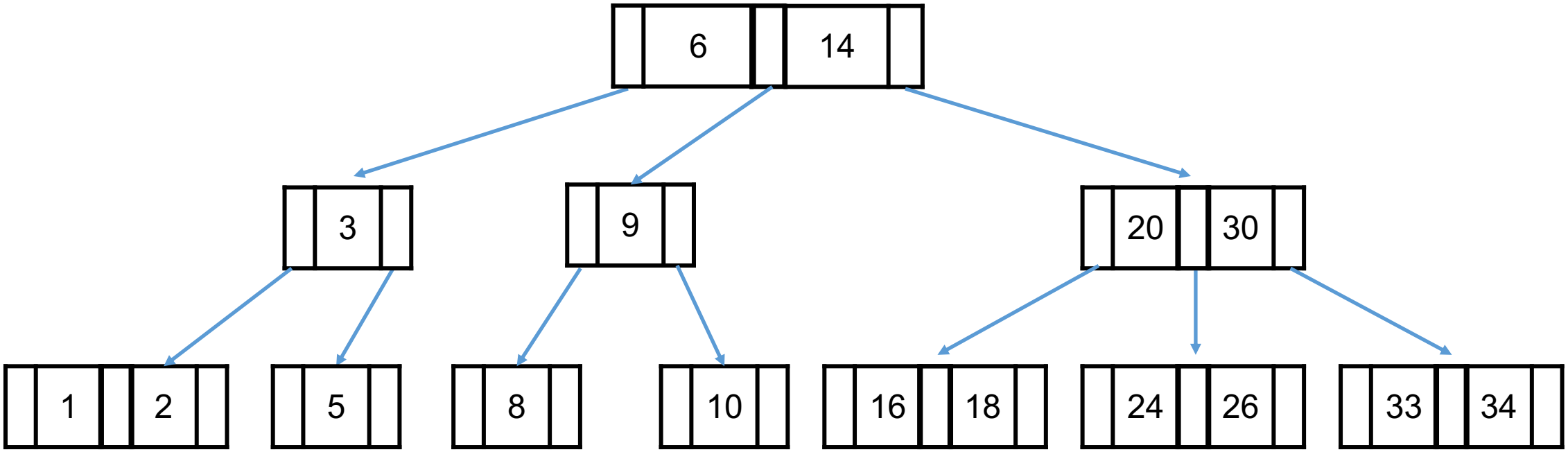
1. 终端节点直接删除
2. 非终端节点，与(前驱/后继)交换，再删除



B 树 – 元素删除(1)

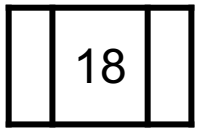
删除：

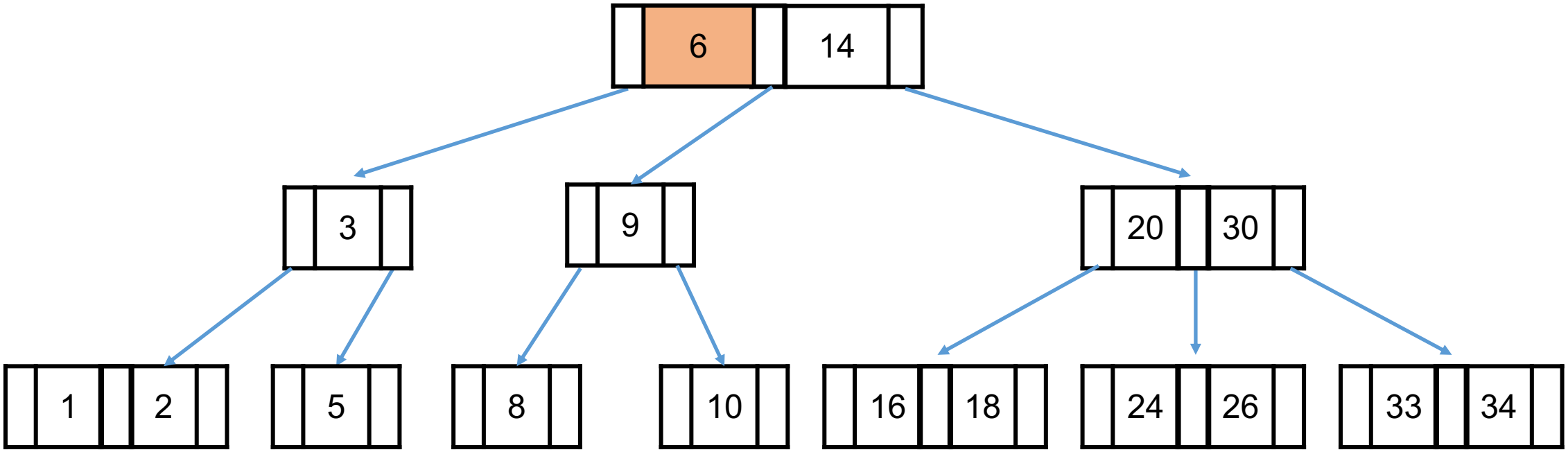




B 树 – 元素删除(1)

删除：





B 树 – 元素删除(1)

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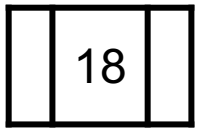
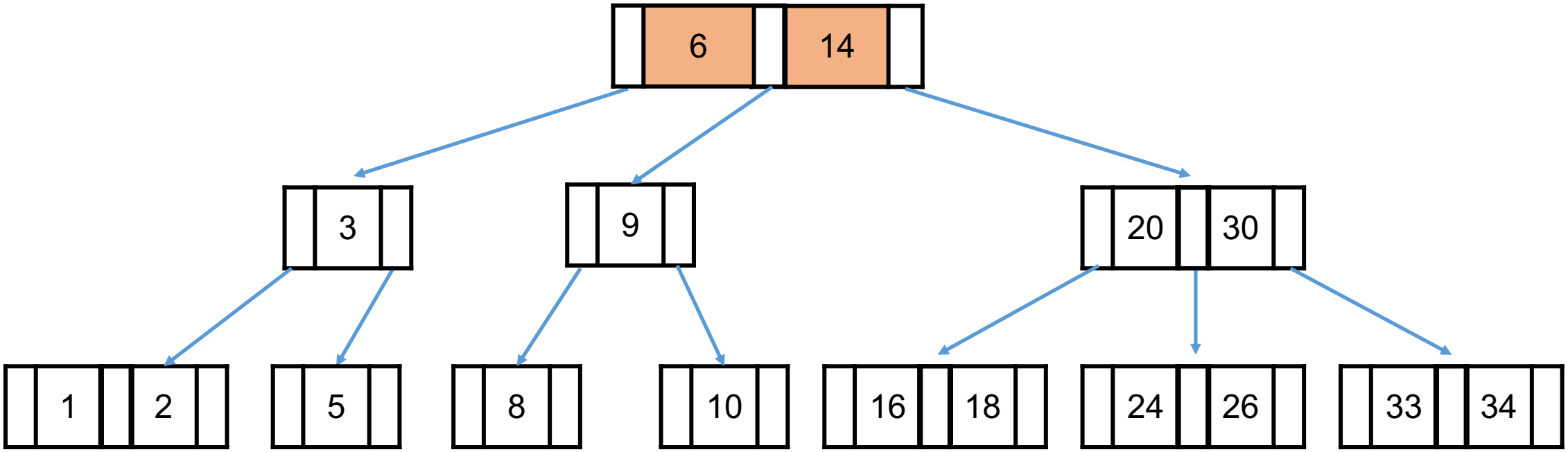
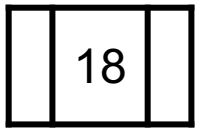
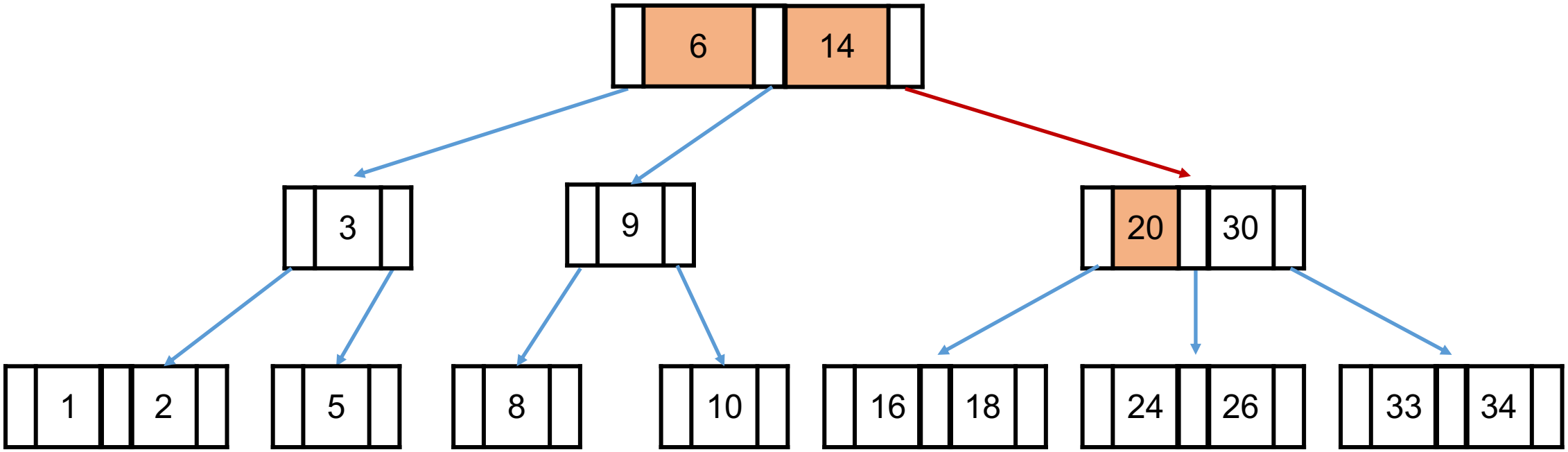




B 树 – 元素删除(1)

删除：





B 树 – 元素删除(1)

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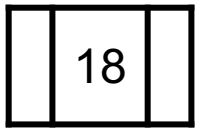

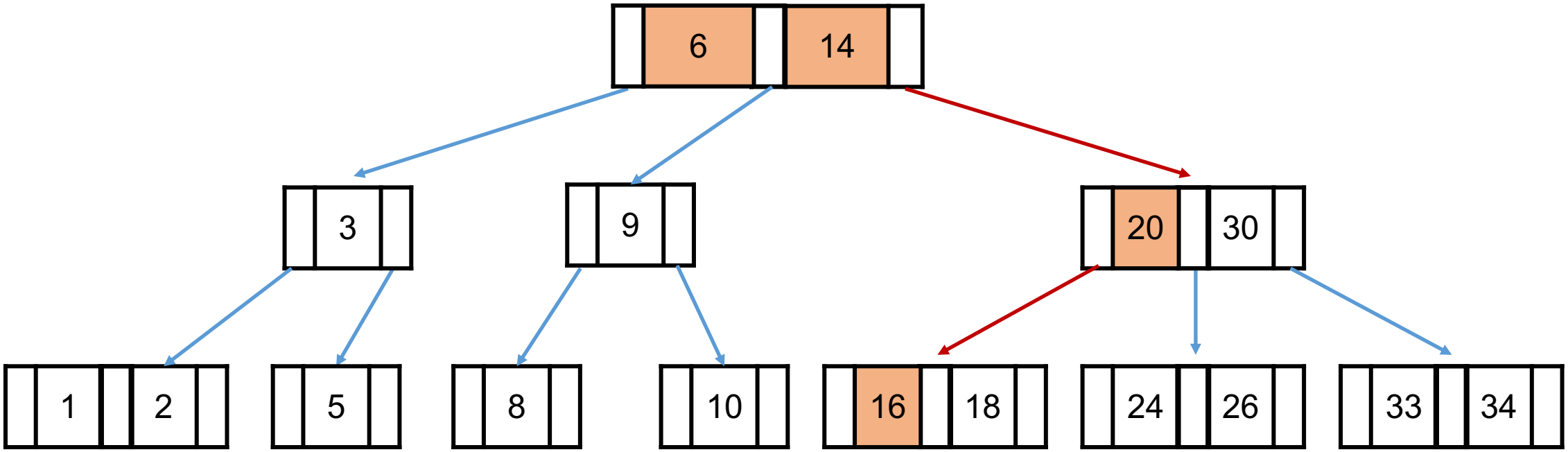

B 树 – 元素删除(1)

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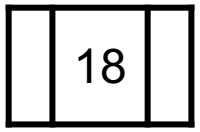

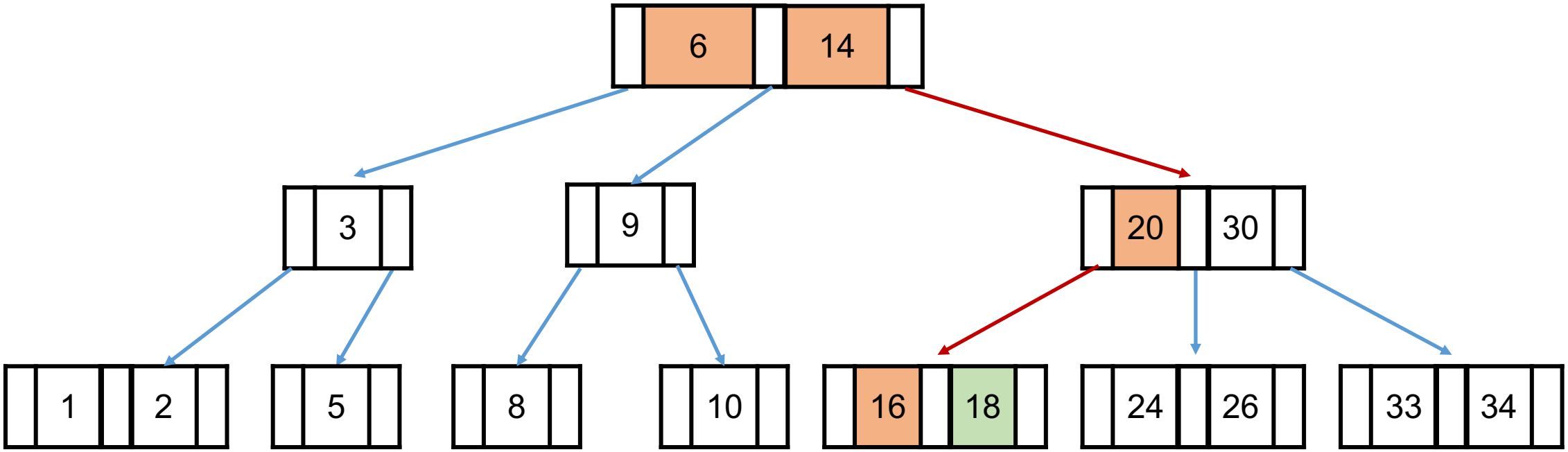

B 树 – 元素删除(1)

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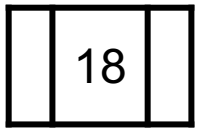
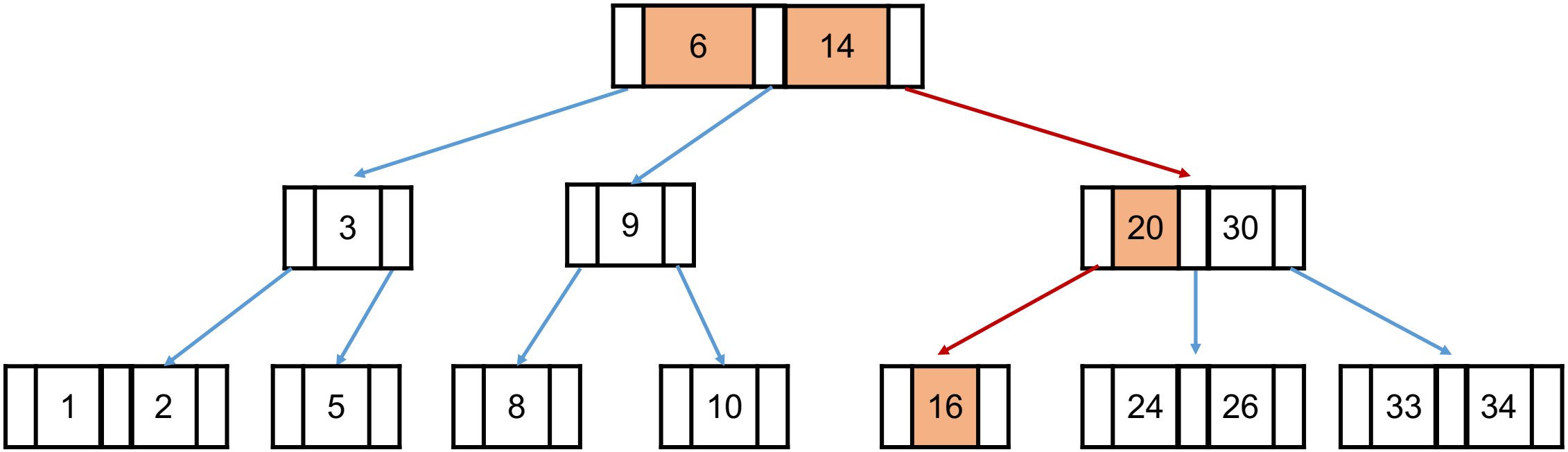
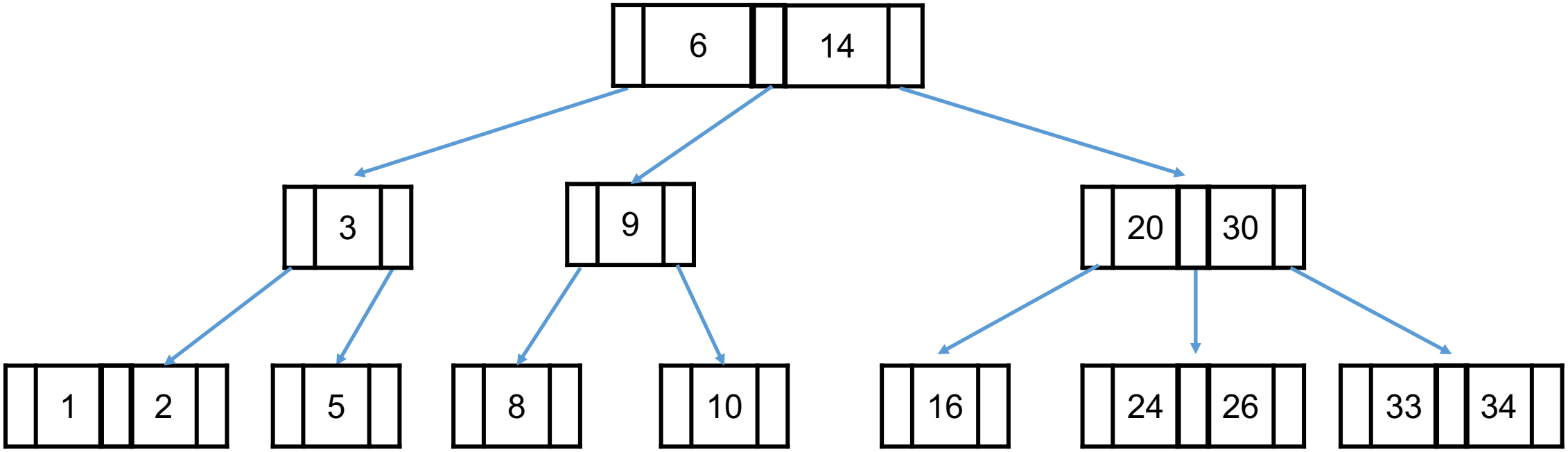


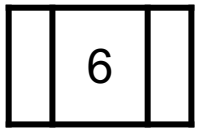


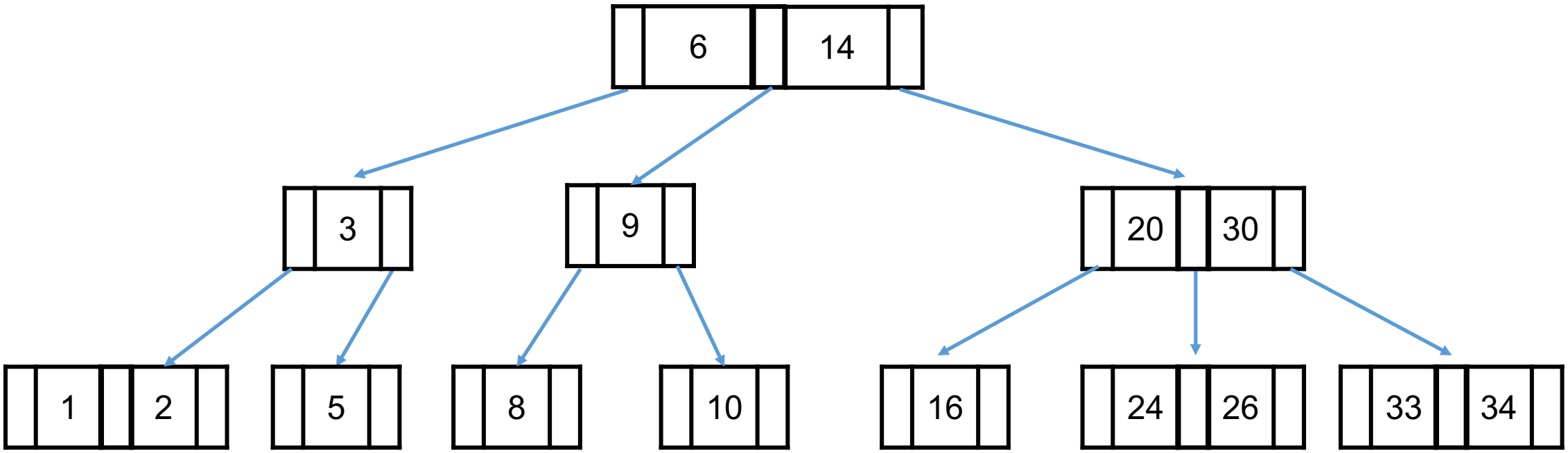
B 树 – 元素删除(1)



B 树 – 元素删除(2)

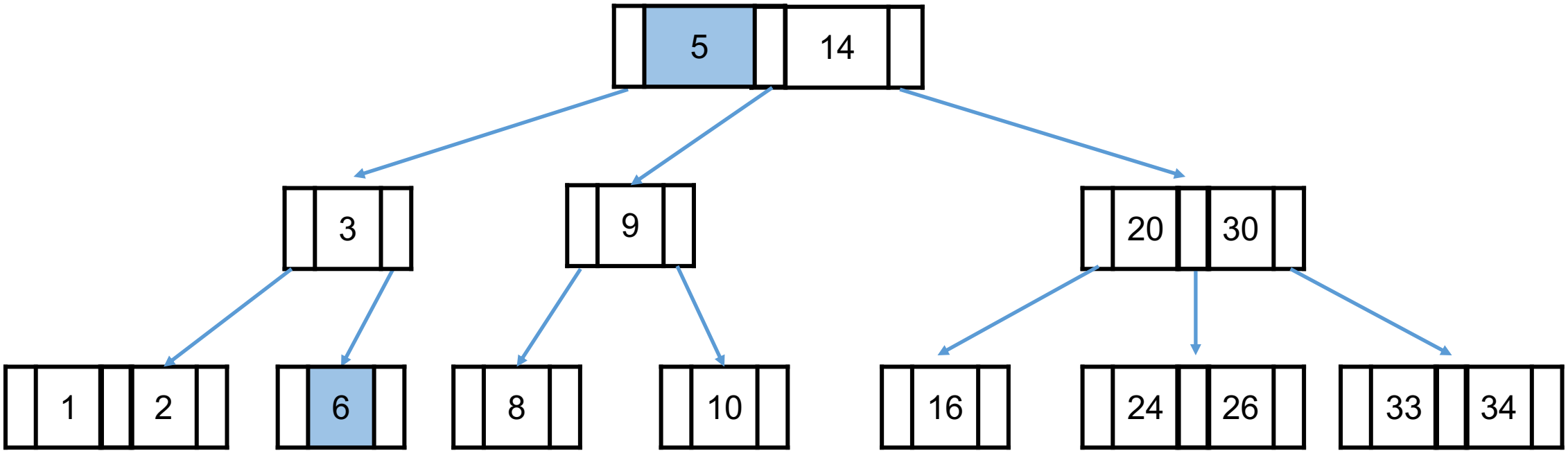
删除：





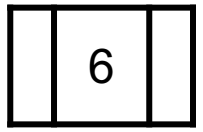
B 树 – 元素删除(2)

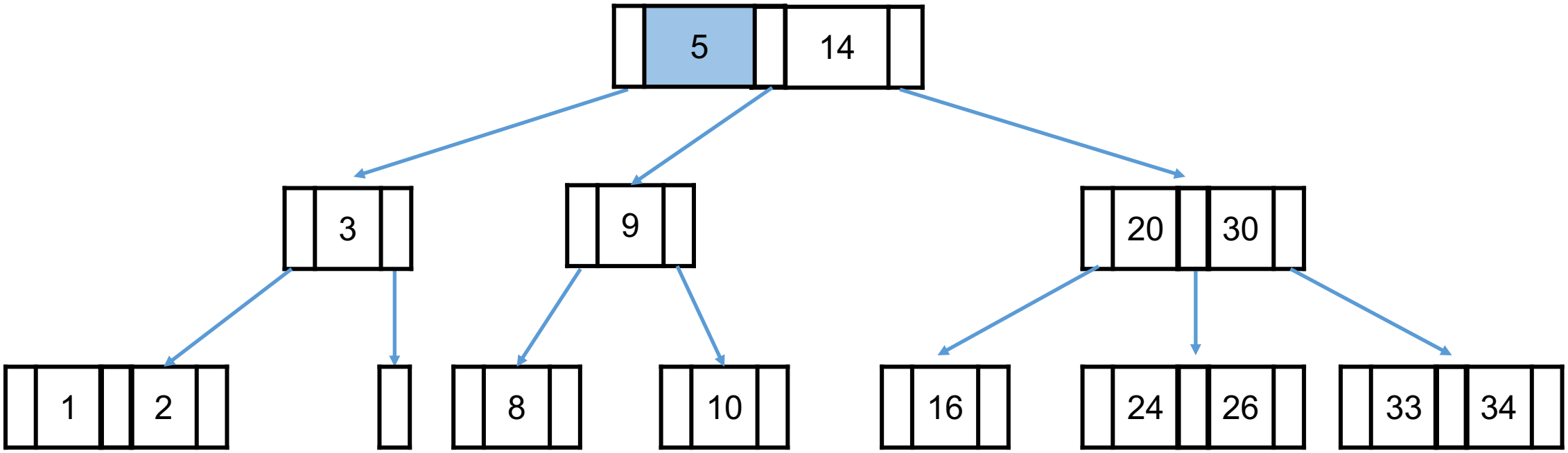
交换 6 和 5



B 树 – 元素删除(2)

删除：





~~1. 元素插入~~

~~2. 插入调整~~

~~3. 元素删除~~

4. 删除调整

~~1. 元素插入~~

~~2. 插入调整~~

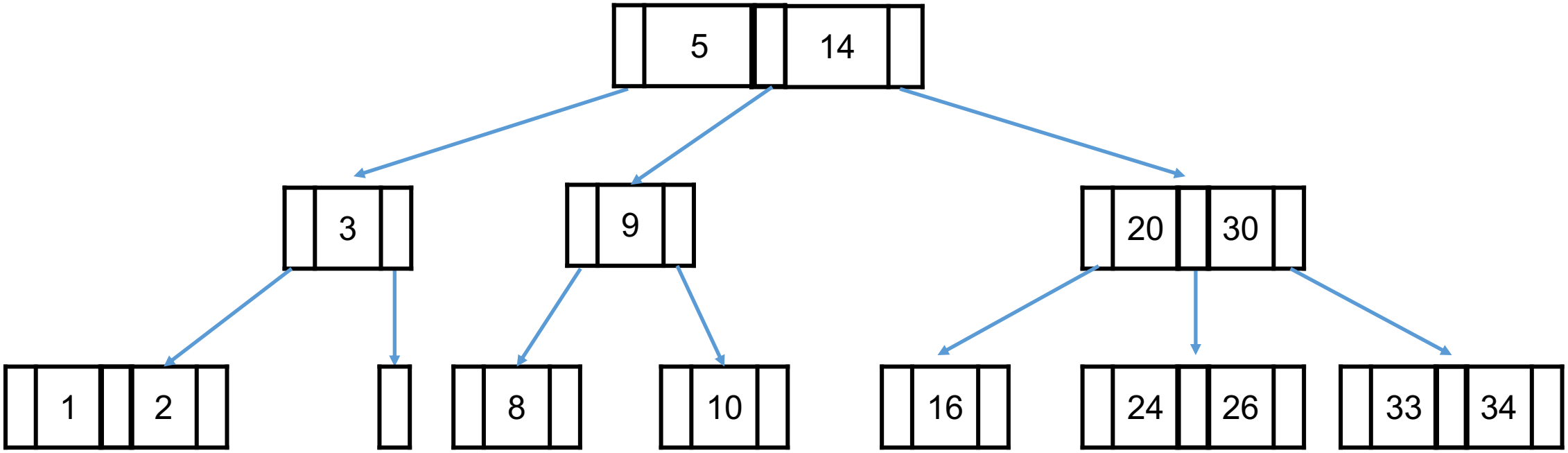
~~3. 元素删除~~

4. 删除调整

B 树 – 删除调整（下溢）

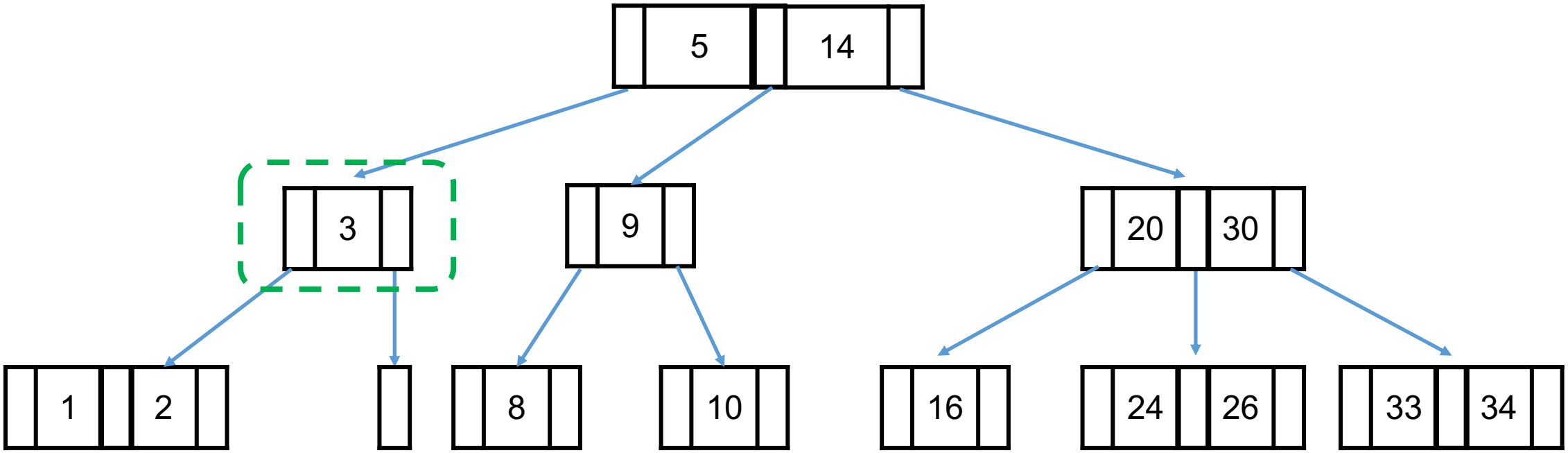
m 阶 B 树的删除调整:

1. 删除调整站在父节点处理，发生在节点关键字数量为 [m/2]-2 时
2. 删除调整的核心操作是: 左旋，右旋与合并



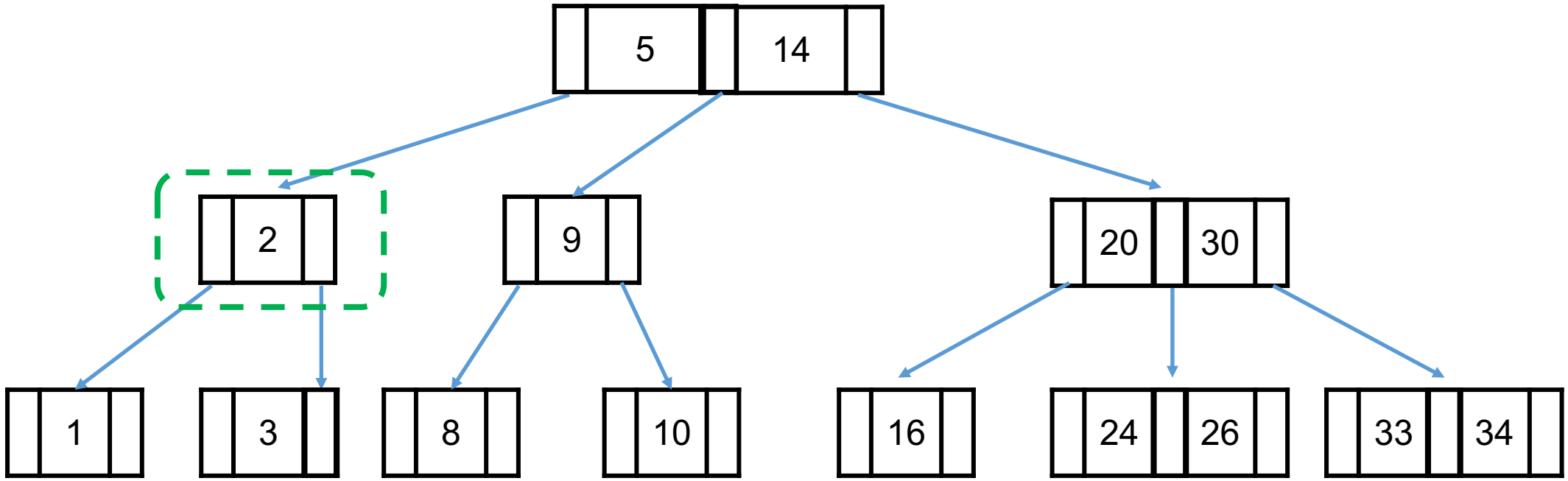
B 树 – 删除调整（下溢）

站在父节点处理

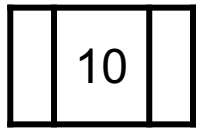


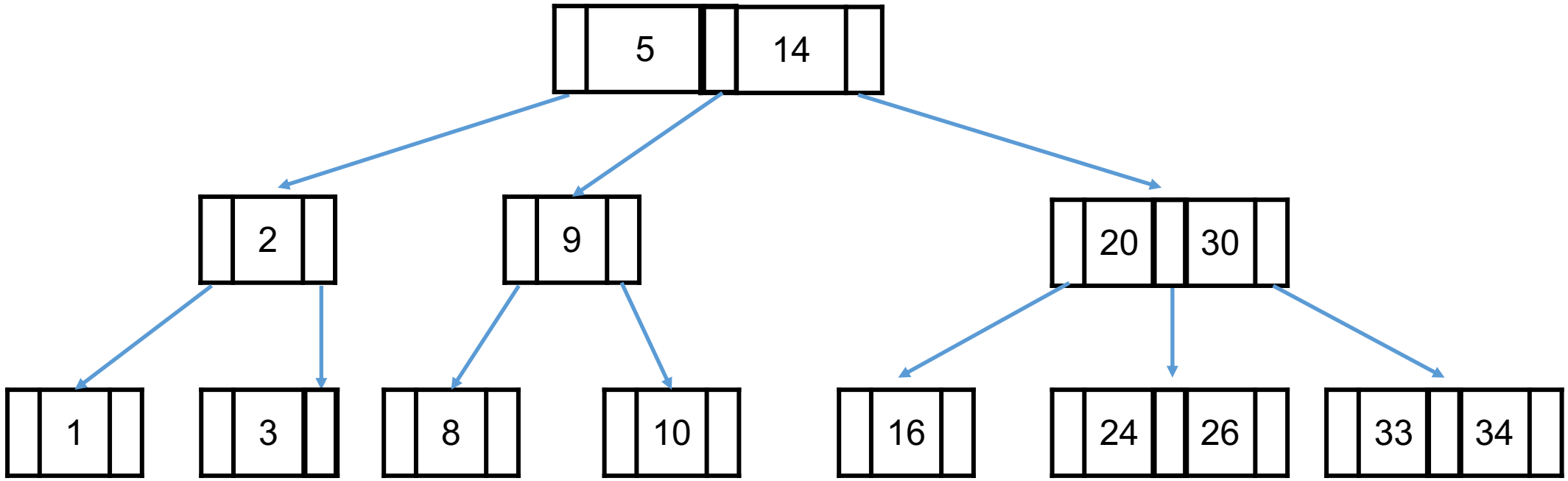
B 树 – 删除调整（下溢）

右旋处理

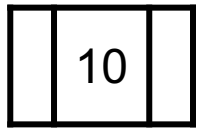


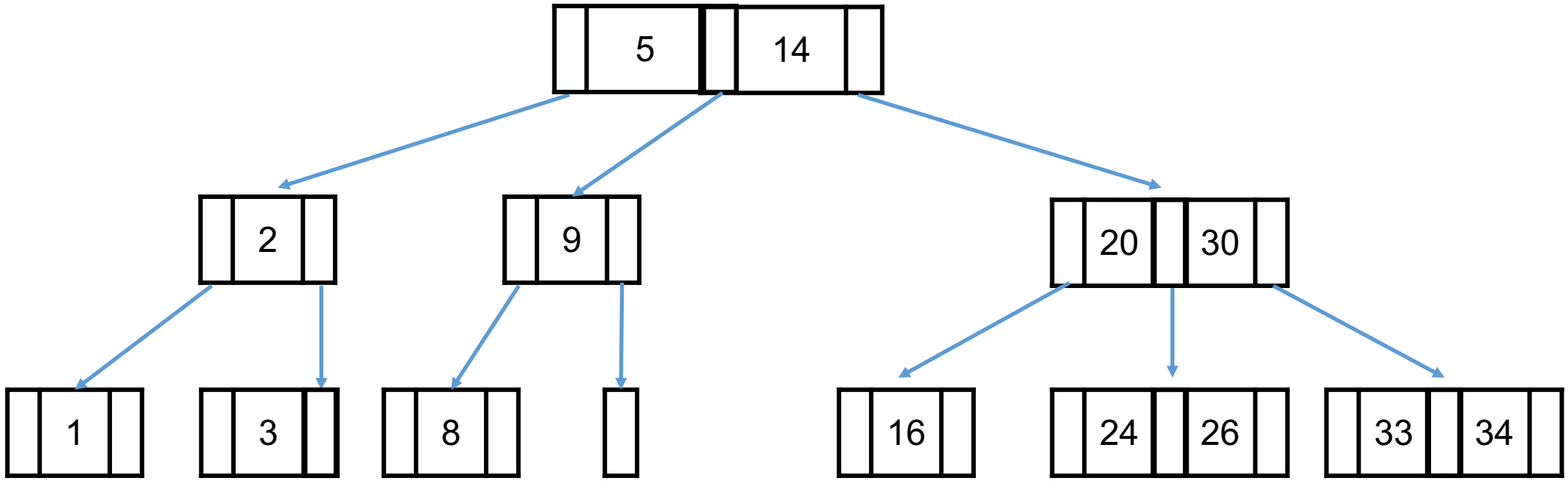
B 树 – 删除调整（下溢）

删除： 



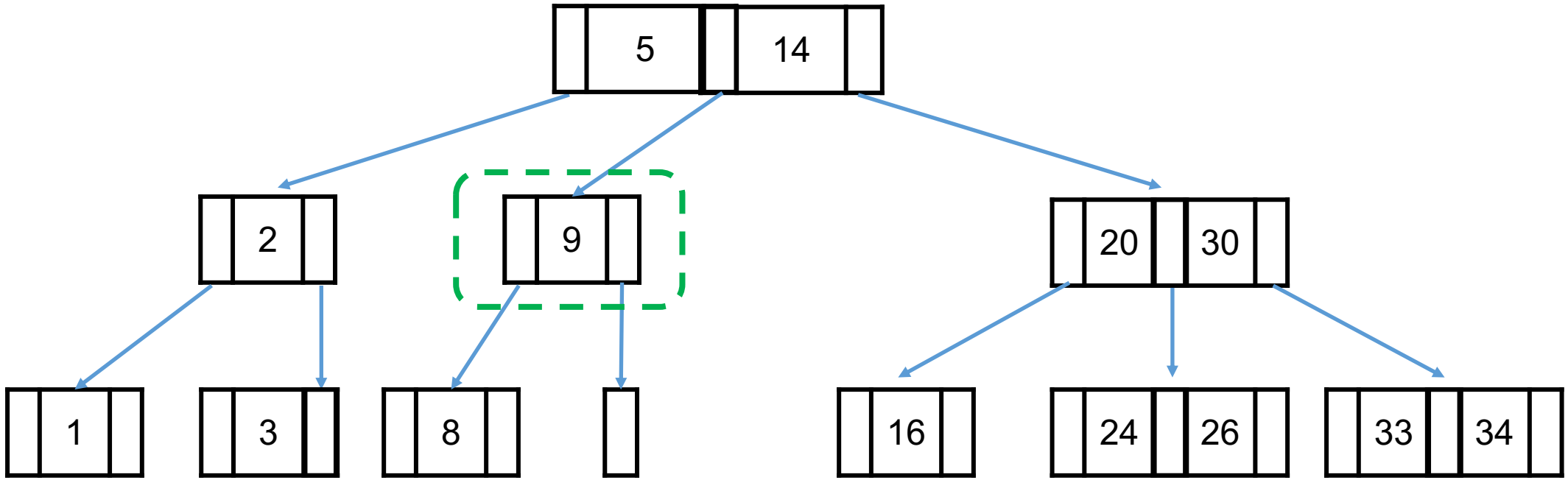
B 树 – 删除调整（下溢）

删除： 



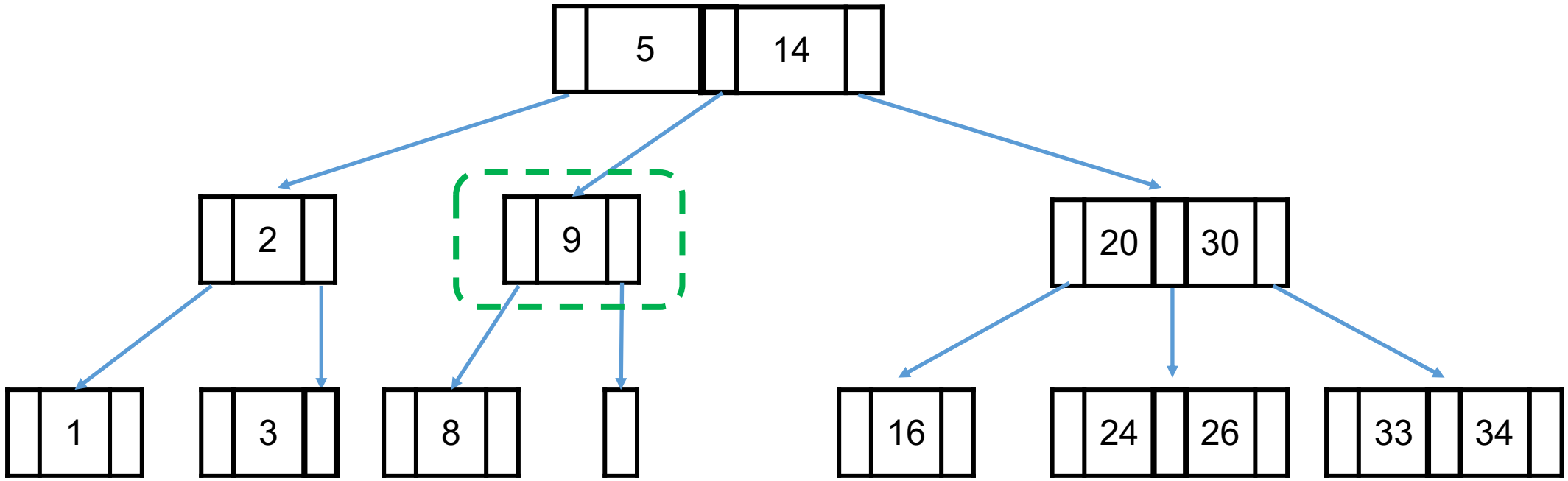
B 树 – 删除调整（下溢）

站在父节点处理



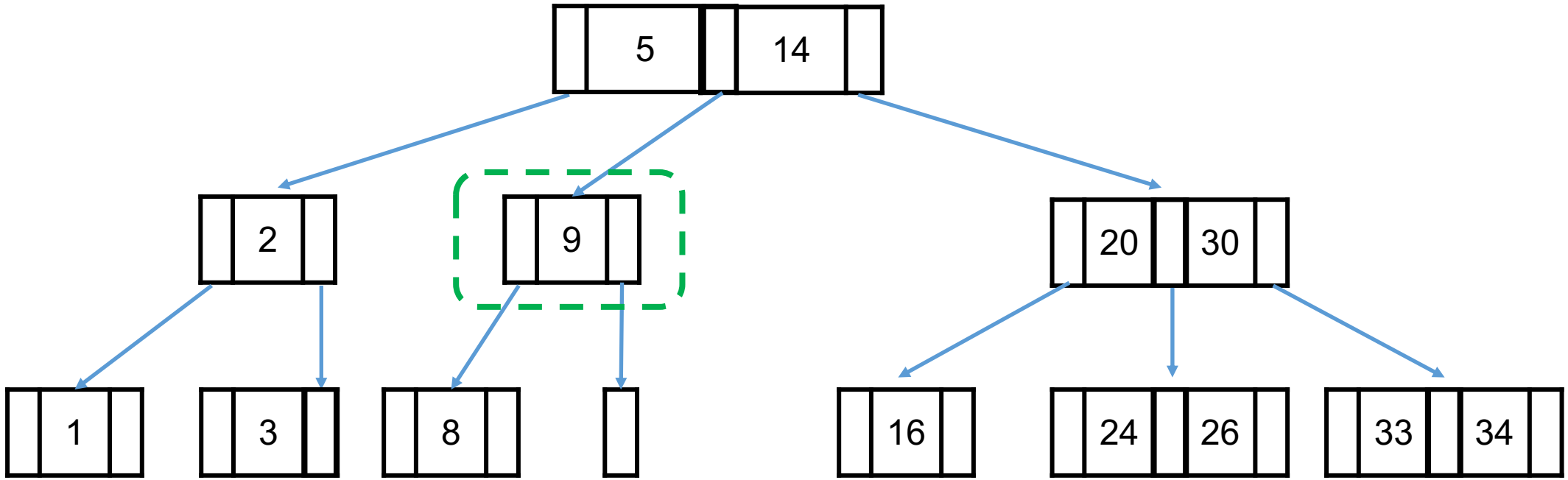
B 树 – 删除调整（下溢）

无法左旋和右旋



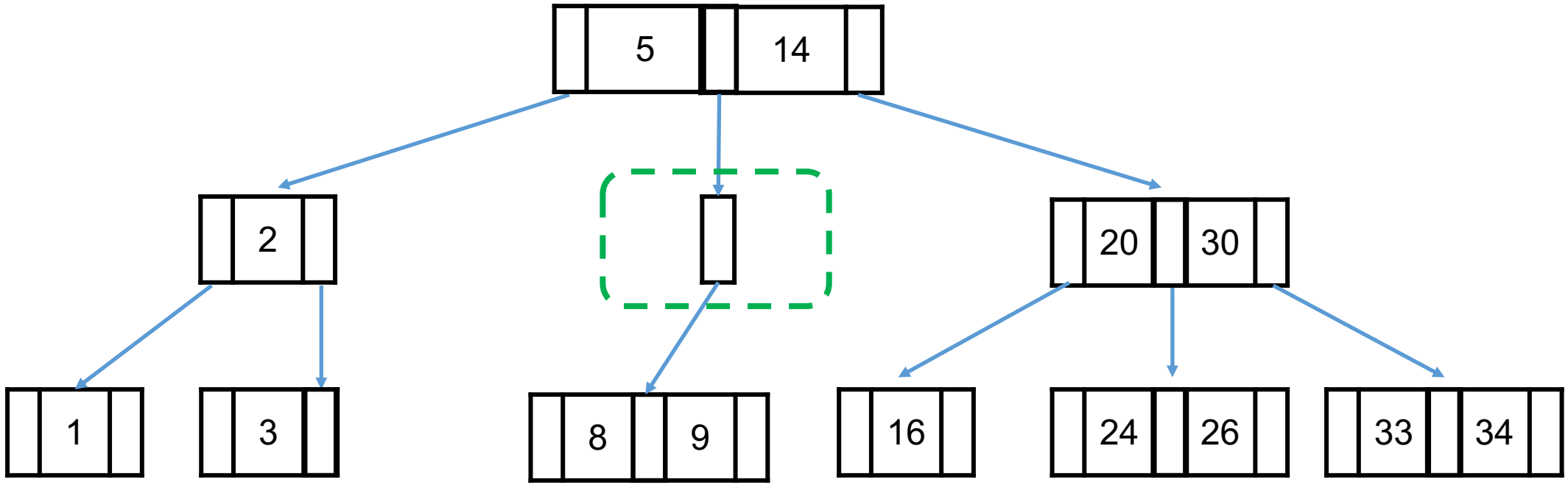
B 树 – 删除调整（下溢）

合并



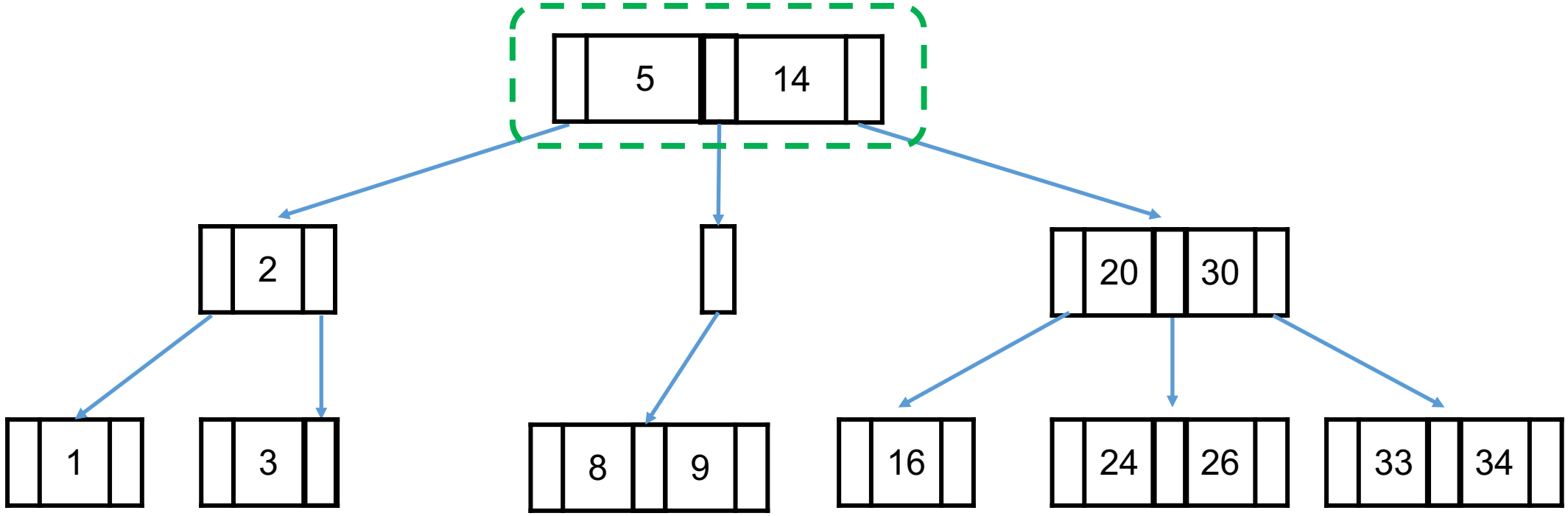
B 树 – 删除调整（下溢）

合并



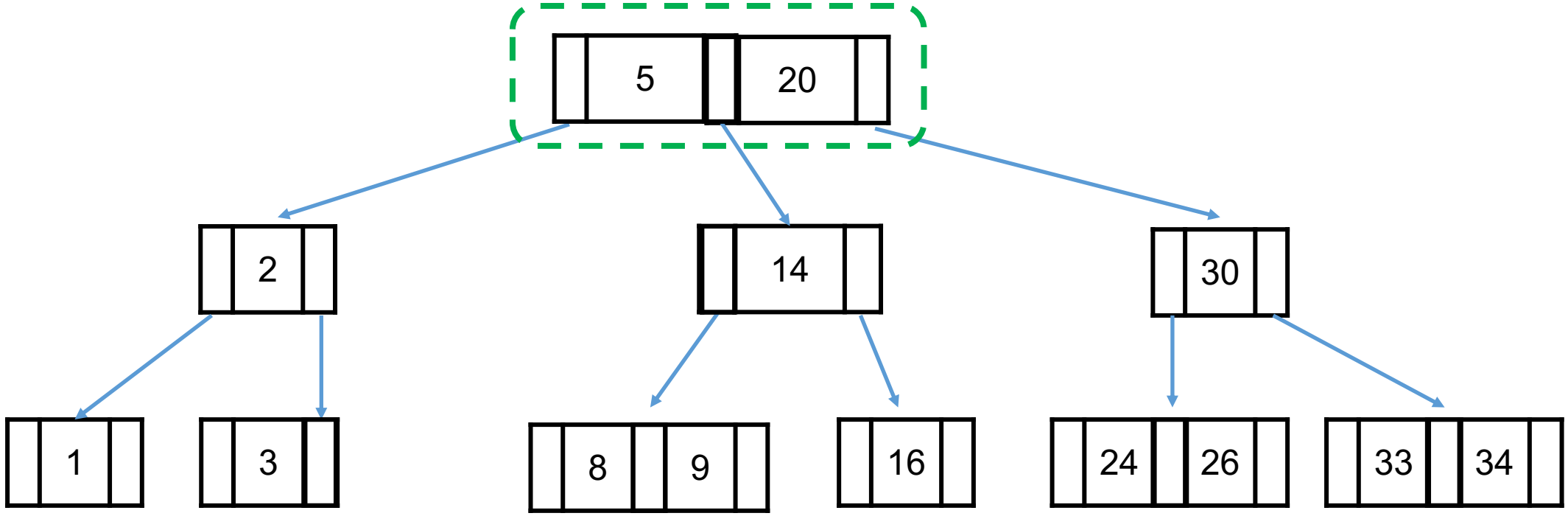
B 树 – 删除调整（下溢）

站在父节点处理



B 树 – 删除调整（下溢）

左旋



1. 元素插入

2. 插入调整

3. 元素删除

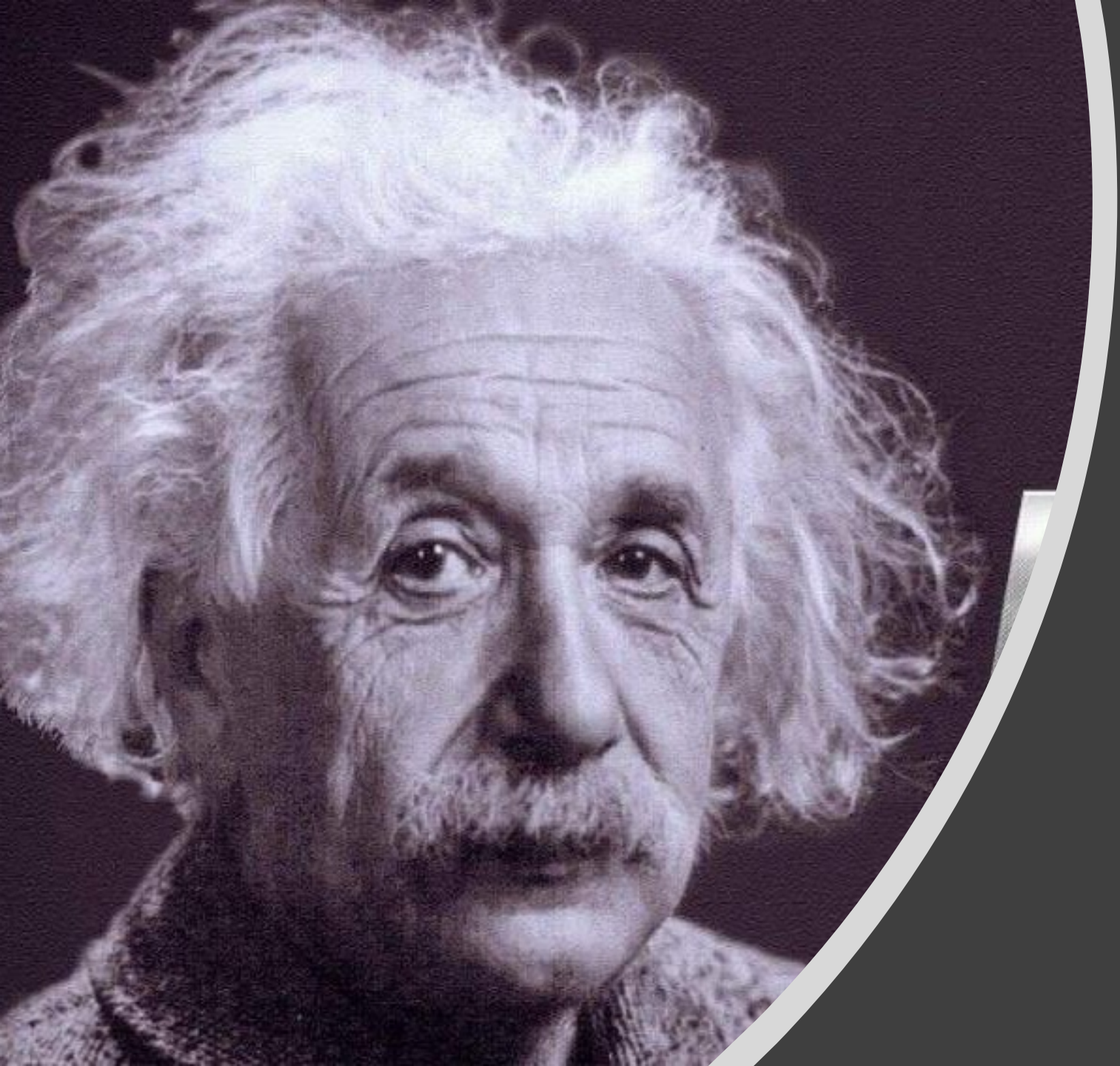
4. 删除调整

B 树 – 总结

m 阶 B 树，定义与操作总结：

1. 树中每个节点，最多含有 m 棵子树
2. 根节点中关键字数量范围 $1 \leq n \leq m-1$
3. 非根结点中关键字数量范围 $\lceil m/2 \rceil - 1 \leq n \leq m-1$
4. 插入调整为了解决『上溢』节点，关键字数量为 m 的节点
5. 删除调整为了解决『下溢』节点，关键字数量为 $\lceil m/2 \rceil - 2$ 的节点
6. 插入调整的核心操作是：节点分裂
7. 删除调整的核心操作是：左旋，右旋与合并

B 树-删除及调整：代码演示



为什么
会出一样的题目？

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第8章-平衡树基础